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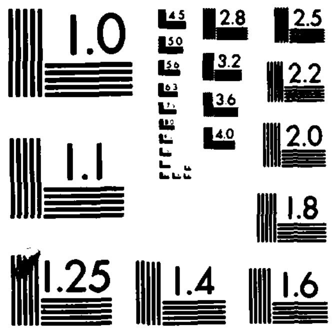
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PARAMETER ESTIMATION

FOR THE

FOUR PARAMETER BETA DISTRIBUTION

THESIS

Bobby R. Treat
Captain, USAF

AFIT/GOR/ENC/83D-6

DEPARTMENT OF THE AIR FORCE
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Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Operations Research

Bobby R. Treat, B.S.

Captain, USAF

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PREFACE

My background consists largely of pure (also known as "useless") mathematics; only here at AFIT have I been introduced to theory subjugated to purpose. Dr. Albert H. Moore showed me probability as elegant measure theory. Lt. Col. James N. Bexfield showed me stochastic process theory with its applications of algebra, characteristic functions, and Laplace Transforms. I am not yet a statistician, for I prefer the tools to the end product.

This thesis is more numerical analysis and software design than statistics, but that reflects the nature of operations research. The work that is described here is but a fraction of the work of a thesis. A thousand validation steps and a hundred crucial decisions can only be hinted at without making the finished product look like a diary. I learned more from the things I did wrong than from those I did right. My thanks to my advisor, Dr. Moore, and my reader, Dr. Joseph P. Cain, for their support.

Bobby R. Treat

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Abstract

This investigation compared six estimation methods for the parameters of the four parameter Beta distribution. The methods used are the full four-moments estimator,.four modified moment estimators, and a modified maximum likelihood estimator.

A closed form solution is derived for the full moment estimator which the author has been unable to find in the literature. The four modified moment estimators are each defined by two moment equations and two equations involving the first and last order statistics. The modified maximum likelihood estimate is defined using the skewness and kurtosis indices to define the shape parameters and then solving for the maximum likelihood estimate of the location parameters, when it exists.

Emphasis is placed on computer application to small samples, n=10 and n=20. Six parent populations are used for sampling purposes, using the six populations (up to symmetry) defined by shape parameter values 0.5, 1, and 2. For sampling purposes, the standard location parameters 0 and 1 are used, but the comparisons should be valid for other location parameters. For each of the twelve sample-size/population combinations, two series of 500 samples each are drawn, for a total of 12,000 samples. All six estimators are applied to each sample. Comparison is done using the logarithm of the Cramer Von Mieses (CVM) unweighted distance between the true and estimated distributions.

Emphasis is also placed on solution techniques and on numerical problems, with descriptions of the way these were handled. The raw computer output, the FORTRAN source code, and summaries of the CVM response are shown, and one estimator is identified as superior to the others.

PARAMETER ESTIMATION

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I. Introduction

An analyst's job often involves fitting a mathematically precise model to an imperfectly known reality and then using the model to simulate reality. A statistician's task, in particular, often amounts to identifying a well-understood probability distribution which adequately describes the variations found in experimental data. Unfortunately, no "best" method exists for performing this identification.

When we calculate the mean and variance of a collection of data and use these two numbers to make statements about the reality which gave rise to the data, we are usually assuming implicitly that the data is adequately modeled as samples from a random variable with a normal (Gaussian) distribution. Sometimes there is good reason to make such an assumption, especially when it appears that the data points are themselves sums of large numbers of independent random effects (whose individual contributions may not be measurable). More often, there is no such theoretical justification, but the assumption is made for convenience and, for limited purposes, it suffices. For instance, when a limited amount of data is available on the response of some dependent variable to a number of factors, it is common to assume an additive model in terms of those factors, with independent and identically distributed normal error terms, so that a classical analysis of variance

can be used to identify significant effects. Yet when a lot of data is available, the additivity, independence, normality, and equal-variance assumptions are too often found to be inadequate. That is, a simple model is commonly used when not enough data exists to refute it, if the model itself is easy to use.

If the data at hand is believed to arise from a continuous distribution with certain properties, it may be attractive to choose a Beta distribution with which to model reality. Each Beta distribution has finite upper and lower bounds and a continuous density function on the open interval between. Different Beta distributions take on a large variety of shapes to choose from, some having one mode or antimode (local maximum or minimum of the density function) and some having a zero of the density function at one bound and a singularity (infinite value) at the other. If the data is believed to arise from a distribution with such properties, it may be sufficient to model it with a well-chosen Beta distribution. There could also be theoretical reasons to expect the true distribution to be Beta (see Chapter II). In either case, the task remains to choose the upper and lower limits and the two shape parameters that determine an appropriate Beta distribution. This task is called parameter estimation.

Parameter estimation always is affected by the kind of penalties we place on different kinds of errors in the estimate and even when appropriate penalties are agreed on, the "best" estimate may be difficult to find. Yet for many families of distributions, good estimators exist. Often the maximum likelihood estimator is used. That is, given a sample $\{x\ (1),\ldots,\ x\ (n)\}$ and a functional form $f_\theta(x)$ for the probability density function (PDF), the parameter (or vector of

parameters) θ is chosen to maximize the likelihood function $L(\theta)$ given by

$$L(\theta) = f_\theta(x(1)) \cdot f_\theta(x(2)) \cdots f_\theta(x(n)) \tag{1}$$

In case $f_{(\mu,\sigma)}(x) = (2\pi\sigma)^{-\frac{1}{2}} \exp\left(-(x-\mu)^2/2\sigma^2\right)$, it turns out that $L(\mu, \sigma^2)$ is maximized when $\mu = \sum_{i=1}^{n} x(i)$ and $\sigma^2 = \sum_{i=1}^{n} (x(i)-\mu)^2/n$. Further analysis shows that this maximum likelihood estimate (MLE) for (μ, σ^2) has some nice properties but that this estimate of σ^2 is biased: it underestimates the true variance, on average. A good unbiased estimator is $\sigma^2 = \sum_{i=1}^{n} (x(i)-\mu)^2/(n-1)$. It is not difficult to derive the above MLE (6:518-520) nor to show that the latter alternative for σ^2 is unbiased (6:397). Considering the other nice properties of these unbiased estimators of μ and σ^2 (15:222-261), including their simplicity, parameter estimation for the normal distribution is essentially no problem.

For the Beta distribution, parameter estimation certainly is a problem. First of all, if all four parameters are allowed to vary, the maximum value of the likelihood function is infinity for any sample and this occurs for infinitely many values of the parameters. Thus no MLE exists for the Beta (see Chapter II). Since methods of finding good unbiased estimators often begin with identifying the MLE, calculating its bias if any, and removing that bias if possible (as in the case of the normal), it is no surprise that no known estimator for the four parameter Beta can so far be considered "best" in any broad sense.

The intent of this thesis is to investigate some estimators for the Beta which are suggested by the work of Cohen and Whitten (2,3,4) on

other distributions. Small samples are emphasized, since for large samples any of these estimators is likely to be adequate. Because the samples are small, attention is largely restricted to moderate shape parameters (from 0.1 to 10.0) and the samples are drawn from even less extreme populations than those allowed as estimates. Nevertheless, six different parent distributions are used, for sampling purposes, which represent the general shapes available in the Beta family.

In Chapter I, the Beta distribution is discussed in general and some notational conventions are established. Next the estimators to be examined are defined. The computer application of the estimators to Beta samples is then discussed in Chapter III, with some attention to the numerical problems that were encountered and resolved. Finally, the results, conclusions, and recommendations for further study are presented.

II. The Beta Distributions

Definitions and Notation

For any shape parameters, $p,q > 0$, the Beta function $\beta(p,q)$ is defined by

$$\beta(p,q) = \int_0^1 x^{p-1} (1-x)^{q-1} \, dx \tag{2}$$

For any location parameters a,b $(a < b)$ and shape parameters $p,q > 0$, the Beta PDF with those parameters is defined by:

$$f(x;a,b,p,q) = \frac{(x-a)^{p-1} (b-x)^{q-1}}{\beta(p,q) \, (b-a)^{p+q-1}} \tag{3}$$

for $a \leq x \leq b$ (zero otherwise). The incomplete Beta function ratio (10:37-56) is defined as

$$I(x;p,q) = \frac{1}{\beta(p,q)} \int_0^x t^{p-1} (1-t)^{q-1} \, dt \tag{4}$$

which is the same as the Beta cumulative density function (CDF) in the case $a=0$, $b=1$.

Throughout the rest of this paper, a,b,p,q will stand for the parameters of a Beta distribution defined by Eq (3) or (4) above. $I(x)$ will be used to mean $I(x;p,q)$ and $f(x)$ will stand for $f(x;a,b,p,q)$. The CDF corresponding to $f(x)$ will be denoted $F(x)$.

Moments

The r^{th} central moment of a random variable $\underline{X}$ will be denoted by μ_r:

$$\mu_1 = E(\underline{X}) \qquad (5)$$

$$\mu_r = E((\underline{X} - \mu_1)^r) \quad (r \geq 1) \qquad (6)$$

The r^{th} central moment of a sample will be denoted by M_r and defined by:

$$M_1 = \sum_{i=1}^{n} x(i)/n \qquad (7)$$

$$M_2 = \sum_{i=1}^{n} (x(i) - M_1)^2 /(n-1) \qquad (8)$$

The indices of skewness and kurtosis are $a_3 = \mu_3 \mu_2^{-3/2}$ and $a_4 = \mu_4 \mu_2{-2}$. For a sample, these shape-factors will be estimated by (8:BESTAT-3,4)

$$A_3 = \frac{\sum_{i=1}^{n} (x(i)-M_1)^3/n}{\left[\sum_{i=1}^{n} (x(i)-M_1)^2/n\right]^{3/2}} \qquad (9)$$

and

$$A_4 = \frac{\sum_{i=1}^{n} (x(i)-M_1)^4/n}{\left[\sum_{i=1}^{n} (x(i)-M_1)^2/n\right]^{2}} \qquad (10)$$

The values of μ_1, μ_2, a_3, and a_4 are given by Johnson and Kotz (10:40) or Bertrand (1:20-22) and are:

$$\mu_1 = a + (b-a)\ p/(p+q) \tag{11}$$

$$\mu_2 = \frac{(b-a)^2\ pq}{(p+q)^2\ (p+q+1)} \tag{12}$$

$$a_3 = \frac{2\ (q-p)\ \sqrt{p+q+1}}{(p+q+2)\ \sqrt{pq}} \tag{13}$$

$$a_4 = \frac{3\ (p+q+1)\ [2(p+q)^2 + pq\ (p+q-6)]}{pq\ (p+q+2)\ (p+q+3)} \tag{14}$$

Order Statistics

Each sample will be considered to be sorted in ascending order, $x(1) \leq x(2) \leq \ldots \leq x(n)$ and so $x(1)$ and $x(n)$ are the first and last order statistics. The expected value of the last order statistic can be computed as (5:7)

$$\begin{aligned} E(\underline{X}(n)) &= \int_a^b x \cdot nF(x)^{n-1}\ dF(x) \\ &= a + (b-a) \int_0^1 x\ I(x)^{n-1}\ dI(x) \end{aligned} \tag{15}$$

The last integral in Eq (15) is the expected value of $\underline{X}(n)$ for the Beta with a=0 and b=1 and will be denoted by $\underline{X}N_{p,q}$. Similarly, we have

$$E\ (\underline{X}(1)) = a + (b-a)\ \underline{X}1_{p,q} \tag{16}$$

and it is not difficult to see that

$$\underline{\overline{X}}1_{p,q} = 1 - \underline{\overline{X}}N_{q,p} \tag{17}$$

Thus, $\underline{\overline{X}}1$ can be computed using $\underline{\overline{X}}N$.

As defined above,

$$\underline{\overline{X}}N_{p,q} = \int_0^1 x\ I(x)^{n-1}\ dI(x)$$

$$= \int_0^1 x\ I(x)^{n-1}\ \frac{x^{p-1}\ (1-x)^{q-1}}{\beta(p,q)}\ dx \tag{18}$$

Because this formula requires computation of both the incomplete Beta ratio and the Beta function in a numerical integration scheme, other equivalent formulae were examined. The first can be obtained by substituting $u = I(x)$ to get

$$\underline{\overline{X}}N_{p,q} = \int_0^1 n\ I^{-1}(u)\ u^{n-1}\ du \tag{19}$$

Substituting $V=u^n$ in this integral gives

$$\underline{\overline{X}}N_{p,q} = \int_0^1 I^{-1}(v^{1/n})\ dv \tag{20}$$

The computation of I^{-1} is very difficult and slow, so the original form is integrated by parts to obtain

$$\underline{\overline{X}}N_{p,q} = (x\ I(x)^n)\Big|_0^1 - \int_0^1 I(x)^n\, dx$$

$$= 1 - \int_0^1 I(x)^n\, dx \qquad (21)$$

Natural Applications of the Beta

The simplest way in which Beta random variables occur is as order statistics from a rectangular distribution (10:38). If a sample of size K is taken from the standard uniform density on the interval (0,1) and if the r^{th} order statistic is selected and called $\underline{\overline{X}}_{r:k}$, then it happens that $\underline{\overline{X}}_{r:k}$ has the Beta distribution with a=0, b=1, p=r, and q=k-r+1.

Another simple case in which Beta variates arise (10:38) is as follows. If $\underline{X}_1^2$ and $\underline{X}_2^2$ are Chi-square random variables with V_1 and V_2 degrees of freedom respectively (not necessarily integers) and if $\underline{X}_1^2$ and $\underline{X}_2^2$ are independent, then the variable $V^2 = \underline{X}_1^2/(\underline{X}_1^2 + \underline{X}_2^2)$ is Beta with a=0, b=1, $p=V_1/2$, and $q=V_2/2$.

The special case p=q=1/2 and a=0, b=1 gives rise to the arc-sine distribution, so named because then

$$I(x) = \frac{2}{\pi} \arcsin \sqrt{x} \qquad (0 \le x \le 1) \qquad (22)$$

This distribution occurs naturally in random walk theory (10:39), and other cases in which $p+q=1$ but $p \neq q$ are called generalized arc-sine distributions.

Beta Distribution Shapes

Beta distributions have a wide variety of shapes (10:41). If $p > 1$ and $q > 1$ then the PDF approaches zero near the location parameters and has a single mode at $x = (p-1)/(p+q-2)$. These distributions are called unimodal. If $p < 1$ and $q < 1$ then the PDF is U-shaped, approaching infinity at both location parameters and with a single antimode at $x = (p-1)/(p+q-2)$. If $(p-1)(q-1) < 0$ then no mode or antimode exists, and the function is J-shaped (or reverse J-shaped). The special case $p=q=1$ is the uniform density and the cases $p=1$, $q=2$ or $p=2$, $q=1$ give linear PDF's. Interchanging p and q is equivalent to reflecting the distribution about $x=1/2$. That is

$$I(x;p,q) = I(1-x;q,p) \tag{23}$$

which was used to derive Eq (17).

III. Estimators for the Beta

The estimators used in this study for the Beta parameters are called modified moment and modified maximum likelihood estimators. Such estimators have been applied by Cohen and Whitten (2,3,4) among others to other distributions although we are aware of no empirical studies for the Beta. Bertrand (1) used a modified moments estimator and a modified maximum likelihood estimator of sorts, but he had problems, and essentially estimated only the shape parameters, after using an extrapolation scheme to determine the location parameters. Bertrand's study was limited to parent distributions for which p and q were both one or larger, and constrained optimization was handled awkwardly.

The six estimators used here are each defined using four non-linear equations chosen out of the four categories discussed below, together with certain constraints, such as $a \leq x(1)$, $b \geq x(u)$, $p > 0$, and $q > 0$. In most cases, a form of the equation is arrived at which is "dimensionless". That is, the terms on both sides of the equation should be scale-independent of the location parameters, at least when the estimates are correct. In each case, an error term is formed from the difference of the two sides of the equation, and in the estimation itself, the goal is to minimize the sum of the squares of four such error terms to determine the best estimate.

Moment Equations

The four moment equations are $\mu_1 = M_1$, $\mu_2 = M_2$, $a_3 = A_3$, and $a_4 = A_4$ and the error terms derived from them are:

$$e_1 = (\mu_1 - M_1)\, M_2^{-2} \tag{24}$$

$$e_2 = (\mu_2 - M_2)/M_2 \tag{25}$$

$$e_3 = a_3 - A_3 \tag{26}$$

$$e_4 = a_4 - A_4 \tag{27}$$

Setting $e_1=e_2=e_3=e_4=0$ and solving for a,b,p, and q gives the full moment estimator (FULLM). Bertrand first determined a and b using an extrapolation method (1:15-17) and then solved $e_1=e_2=0$ (using the biased sample variance however) to determine p and q. He then attempted an iterative solution of the four moment equations using this as a starting point, but found that the numerical routine used to do this, ZSCNT (from IMSL (8)), almost always attempted negative values of p and q and so this failed.

The approach of this study to the problem was different. First of all, Johnson and Kotz (10:41-44) give a closed form solution of the four moment equations, so that iteration seemed unnecessary. Unfortunately, this closed form solution was found to be in error, but fortunately again, this author was able to derive the correct closed form solution. This solution may well be in the literature already, but we have not found it there. The derivative follows.

The original equations are:

$$M_1 = a + \frac{(b-a)\,p}{p+q} \tag{28}$$

$$M_2 = \frac{pq\ (b-a)^2}{(p+q)^2\ (p+q+1)} \tag{29}$$

$$A_3 = \frac{2(q-p)\ \sqrt{p+q+1}}{(p+q+2)\ \sqrt{pq}} \tag{30}$$

$$A_4 = \frac{3(p+q+1)\ [2(p+q)^2 + pq(p+q-6)]}{pq\ (p+q+2)\ (p+q+3)} \tag{31}$$

Substituting r = p+q and q = r-p in the last equation gives

$$A_4 = \frac{3(r+1)\ [2r^2 + p(r-p)\ (r-6)]}{p(r-p)\ (r+2)\ (r+3)} \tag{32}$$

$$= \frac{6\ (r+1)\ r^2}{p\ (r-p)\ (r+2)\ (r+3)} + \frac{3\ (r+1)\ (r-6)}{(r+2)\ (r+3)} \tag{33}$$

from which we obtain

$$\frac{6\ (r+1)\ r^2}{p(r-p)\ (r+2)\ (r+3)} = A_4 - \frac{3\ (r+1)\ (r-6)}{(r+2)\ (r+3)} \tag{34}$$

and so

$$p\ (r-p) = \frac{6(r+1)\ r^2}{(r+2)\ (r+3)}\ [A_4 - \frac{3\ (r+1)\ (r-6)}{(r+2)\ (r+3)}\ ]^{-1} \tag{35}$$

$$= \frac{6\ (r+1)\ r^2}{A_4\ (r+2)\ (r+3) - 3\ (r+1)\ (r-6)} \tag{36}$$

This is the quadratic in p,

$$p^2 - rp + \frac{6\ (r+1)\ r^2}{A_4\ (r+2)\ (r+3) - 3\ (r+1)\ (r-6)} = 0 \qquad (37)$$

whose solution is

$$p = 1/2\ (r \pm \sqrt{r^2 - \frac{24\ (r+1)\ r^2}{A_4\ (r+2)\ (r+3) - 3\ (r+1)\ (r-6)}}\ ) \qquad (38)$$

$$= \frac{r}{2}\ (1 \pm \sqrt{1 - \frac{24\ (r+1)}{A_4\ (r+2)\ (r+3) - 3\ (r+1)\ (r-6)}}\ ) \qquad (39)$$

Note then that

$$(r-2p)^2 = r^2\ (1 - \frac{24\ (r+1)}{A_4\ (r+2)\ (r+3) - 3\ (r+1)\ (r-6)}\ ) \qquad (40)$$

Now cross-multiplying and squaring both sides in Eq (30) gives

$$A_3^2\ (r+2)^2\ p(r-p) = 4\ (r-2p)^2\ (r+1) \qquad (41)$$

Using Eq (36) to substitute on the left and Eq (40) to substitute on the right and dividing out like terms, and then multiplying by

$$[A_4(r+2)\ (r+3) - 3\ (r+1)\ (r-6)]/4$$

reduces Eq (41) to

$$6A_3^2\ (r+2)^2/4 = A_4\ (r+2)\ (r+3)-3(r+1)\ (r-6)-24\ (r+1) \tag{42}$$

$$= (r+2)\ [A_4\ (r+3) - 3(r+1)] \tag{43}$$

Then simplification gives

$$3/2\ A_3^2\ (r+2) = A_4\ (r+3) - 3\ (r+1) \tag{44}$$

and so

$$r = \frac{6\ (A_4 - A_3^2 - 1)}{6 + 3A_3^2 - 2A_4} \tag{45}$$

Eq (39) was derived entirely from Eq (31), in which p and q appear symmetrically, so the right side of Eq (39) is also the solution for q (with the signs reversed). Thus, we have two solutions to choose from. Either solution simultaneously solves Eqs (31) and (41). But in Eq (41) and those derived from it, the sign of A_3 has been lost and it is precisely the sign of A_3 which we need, looking at Eq (30), to resolve the ambiguity. Clearly A_3 and q-p must have the same sign and we choose the sign in Eq (39) accordingly so that $p < q$ if $A_3 > 0$ and $p > q$ if $A_3 < 0$. When $r > 0$ (as it must be) this means a '+' sign in Eq (39) if $A_3 \leq 0$ and a '-' sign otherwise.

The solution given by Johnson and Kotz involves an intermediate factor exactly as given in Eq (45) but a quite different expression in place of Eq (39), with the sign determined in the same way.

The full moment estimator is completed by solving Eqs (28) and (29) for a and b. This method cannot fail, unless Eq (45) results in $r \leq 0$. It is not clear whether that can occur at all, but it was never observed in this study. The estimates a and b can be inconsistent, however, in that $a > x(1)$ and/or $b < x(n)$. In that case, the inconsistent parameter is set equal to the bound on it, and iteration is performed to minimize $e_1^2 + e_2^2 + e_3^2 + e_4^2$ as a function of all four parameters, with the proper constraints.

To do all of the constrained optimization involved in this study, upper and lower limits were set for each parameter (0.1 and 10.0 for the shape parameters) and a method was used which was briefly mentioned in the IMSL Reference Manual (8:ZXMWD-2). Minimizing a function $f(x_1,\ldots,x_k)$ subject to $a_i \leq x_i \leq b_i$ is equivalent to minimizing the function

$$g(t_1,\ldots,t_k) = f(a_1+(b_1-a_1)\sin^2 t_1,\ldots,a_k+(b_k-a_k)\sin^2 t_k) \tag{46}$$

where now the t_i are unconstrained. Thus, the above transformation is done so that IMSL routines can be used successfully for constrained optimization problems.

Finally, since it seemed that all of the methods had more difficulty with the shape parameters than the location parameters, Eq (25) was changed to

$$e_2 = (\mu_2-M_2)\, M_2^{-1/2} \tag{25'}$$

to give that equation less weight in the estimations. Over the range of populations used, this is roughly equivalent t^ di 'ding the right

side of Eq (25) by four. The difference only matters in cases where iteration is used, but in retrospect use of a constant weight factor might have been better.

Modified Maximum Likelihood Estimation

The log-likelihood function $\theta(a,b,p,q)$, given a sample $x(1),\ldots,x(n)$, is

$$\theta = (p-1)\sum_{i=1}^{n} \ln(x(i)-a) + (q-1)\sum_{i=1}^{n} \ln(b-x(i))$$

$$- n \ln \beta(p,q) - n(p+q-1)\ln(b-a) \tag{47}$$

To maximize $\theta(a,b,p,q)$ we would ordinarily set its partial derivatives equal to zero:

$$\theta_a = (1-p)\sum_{i=1}^{n} (x(i)-a)^{-1} + \frac{n(p+q-1)}{b-a} = 0 \tag{48}$$

$$\theta_b = (q-1)\sum_{i=1}^{n} (b-x(i))^{-1} - \frac{n(p+q-1)}{b-a} = 0 \tag{49}$$

$$\theta_p = \sum_{i=1}^{n} \ln\left(\frac{x(i)-a}{b-a}\right) - n\frac{\beta_p(p,q)}{\beta(p,q)} \tag{50}$$

$$\theta_q = \sum_{i=1}^{n} \ln\left(\frac{b-x(i)}{b-a}\right) - n\frac{\beta_q(p,q)}{\beta(p,q)} \tag{51}$$

The function θ is in fact unbounded since if $0 < p < 1$ then $\theta(a,b,p,q)$ approaches infinity as a approaches the minimum of the sample from below.

Thus, any such p, a = x(1), and any choice of b and q could be considered a maximum likelihood estimate. Therefore, no meaningful maximum likelihood estimate actually exists, when all four parameters can vary.

It is of interest to consider when Eqs (48) and (49) might have solutions when p and q are known, so we will examine the implications of those two equations. Adding Eqs (48) and (49) together gives

$$(p-1) \sum_{i=1}^{n} (x(i)-a)^{-1} = (q-1) \sum_{i=1}^{n} (b-x(i))^{-1} \qquad (52)$$

from which the condition $(p-1)\ (q-1) \geq 0$ is evident. From this it is also clear that if $(p-1)\ (q-1) = 0$ then $p=q=1$, but then Eqs (48) and (49) cannot hold. Thus $\theta_a=\theta_b=0$ implies that $(p-1)\ (q-1) > 0$. If $p < 1$ then also $q < 1$ and any a and b which satisfy $\theta_a=\theta_b=0$ in that case would be minimum likelihood estimates of a and b rather than maximum likelihood estimates, so solving these equations is of no benefit unless $p > 1$ and $q > 1$.

Combination of some of the maximum likelihood equations with other equations to obtain a system which does have a meaningful solution can be called modified maximum likelihood estimation. In this study, the equations $e_e=e_4=\theta_a=\theta_b=0$ gives such an estimator, called MMLE in this study. When $e_3=e_4=0$ results in $p \leq 1$ or $q \leq 1$, the MMLE estimate is simply the FULLM estimate, for convenience.

For any $p > 1$ and $q > 1$, useful bounds were derived for a and b solving $\theta_a=\theta_b=0$. The derivation is simple though tedious and will not be included here. These bounds can be expressed in the form:

$$-1 < \frac{a-x(1)}{(p-1)\ (x(n)-x(1))} < \frac{-1}{(n-1)\ (p+q-1) + 1} \tag{53}$$

and

$$\frac{1}{(n-1)\ (p+q-1) + 1} < \frac{b-x(n)}{(q-1)\ (x(n)-x(1))} < 1 \tag{54}$$

Slightly better bounds can be derived but these were sufficient for the constrained optimization methods used.

Modified Moment Estimation

Replacing some of the four moment equations with other suitable equations and solving the resulting system can be called modified moment estimation (2,3,4). Thus, the estimator MMLE described above is also a modified moment estimator. The supplementary equations used in this study involve the first and last order statistics. The first two are:

$$e_5 = \frac{x(1)-a}{x(n)-x(1)} - \frac{\bar{\underline{X}}1_{p,q}}{\bar{\underline{X}}N_{p,q}-\bar{\underline{X}}1_{p,q}} = 0 \tag{55}$$

$$e_6 = \frac{x(n)-a}{x(n)-x(1)} - \frac{\bar{\underline{X}}N_{p,q}}{\bar{\underline{X}}N_{p,q}-\bar{\underline{X}}1_{p,q}} = 0 \tag{56}$$

These are derived from setting $E(\underline{X}(1)) = x(1)$ and $E(\underline{X}(n))=x(n)$.

The second two equations are:

$$e_7 = I\left(\frac{x(n)-a}{b-a}\right) - \frac{n}{n+1} = 0 \tag{57}$$

$$e_8 = I\left(\frac{x(1)-a}{b-a}\right) - \frac{1}{n+1} = 0 \tag{58}$$

These are derived from the equations:

$$E\left(I(\underline{X}(n))\right) = I\left(x(n)\right) \tag{59}$$

$$E\left(I(\underline{X}(1))\right) = I\left(x(1)\right) \tag{60}$$

Similar equations were used by Cohen and Whitten because they contain a lot of information about location parameters. Their studies involved distributions with only one location parameter and so only the equations involving the first order statistic were used. Their applications also have not included any four-parameter families to our knowledge.

In this study, four different modified moment estimators are used (not including MMLE) and these are:

$$\text{MME1:}\quad e_1=e_2=e_7=e_8=0 \tag{61}$$

$$\text{MME2:}\quad e_1=e_2=e_5=e_6=0 \tag{62}$$

$$\text{MME3:}\quad e_3=e_4=e_7=e_8=0 \tag{63}$$

$$\text{MME4:}\quad e_3=e_4=e_5=e_6=0 \tag{64}$$

Many other combinations are possible, and an exhaustive study was impractical, but the six estimators above are thought to be representative of the estimators available and sufficient to point the direction to other promising estimators.

IV. Computer Applications

Study Design

The six estimates defined in Chapter III were compared at sample sizes n=10 and n=20 using six different parent Beta distributions for sampling. Those distributions all use a=0 and b=1 but the estimation routines were not allowed to make use of that fact nor even to start with any magnitudes in mind for the location parameters, other than those derived from the sample. The criteria were kept dimensionless as near as possible (see Chapter III) and so it is felt that extrapolation of these results to other location parameters is valid. The six populations were chosen by using the levels 0.5, 1.0, and 2.0 for each parameter and using only $p \leq q$ because of symmetry.

For each combination of the two sample sizes and six parent distributions used, two different series of 500 samples were generated, for a total of 1000 samples. Two random number seeds were used, each to generate 500 samples, rather than a single series of 1000 samples. This was done partly as insurance against a badly chosen seed number and partly because some of the estimators were so slow that two separate runs of 500 samples each were easier to accomplish than a single run of 1000 samples. The total of 1000 samples was chosen arbitrarily on the advice of the advisor. Thus, 12,000 samples were generated and all six estimators were used on each sample.

Output consisted of aggregate data for each series of 500 samples, for each combination of sample size, seed number, and population (see Appendix A). The estimates were to be compared using the Cramer Van Mieses

unweighted distance measure (1:10-11) defined by:

$$\text{CVM} = \int_0^1 [I(x)-F(x)]^2 \, dI(x) \qquad (65)$$

where I(x) is the true CDF and F(x) is the CDF determined by the estimate values.

Other measures were considered, including the mean-square errors in the individual parameters, the sum of these four mean-square errors, or the Anderson-Darling distance measure (1:10-11). The mean-square errors were found to be very highly variable and would have been greatly affected by the true scale in any case, so those are reported in the raw data but not used for comparison. The Anderson-Darling measure was first considered because it would penalize errors in the location parameters more directly than other measures, but it was soon realized that too often that measure would be undefined. In fact, it would be infinite unless the estimated location parameters lie strictly within the true support of the parent distribution. The CVM is both dimensionless and always defined.

Solution Techniques

Like Bertrand's thesis (1) this study made heavy use of IMSL routines (8) to generate Beta samples, to calculate I(x), to sort samples, to generate sample moments, and also to solve the nonlinear equations represented by the estimators in Chapter III. Several equation-solving routines were tried before adequate performance was obtained. ZSCNT was too unstable and seldom yielded a solution. ZXMIN was too slow in cases (like MME1

and MME2) where function evaluation was very slow. ZSPOW was finally found to be adequate and was used for all of the iterative solutions (8:ZSPOW:1-3). Although ZXMWD (8:ZXMWD:1-3) was also too slow (since it uses ZXMIN many times), the description of ZXMWD led to the transformation (46) which worked quite well for these estimators.

Thus, ZSPOW was used to perform unconstrained optimization in terms of the transformed variables, to accomplish constrained optimization in terms of the original variables. A tendency existed for the transformed variables to get so large that the transformation and its inverse could not be performed accurately, so a penalty function was added to that being minimized, to discourage large values while leaving function values unchanged for more moderate values of the transformed variables. (See subroutines SHAPE, RESHPE, PENLTY, FCN2, and FCN4 in Appendix B).

Computation Problems

Several significant computational problems were encountered and resolved. The first was the computation of $\underline{\overline{X}}N_{p,q}$. Eq (18) was not used because no direct way of computing $\beta(p,q)$ was available although computation of I(x) using MDBETA (8) was straightforward. It was also felt that the resulting unbounded integrand (for $p < 1$ or $q < 1$) would cause problems in numerical integration. Eq (20) was tried and consistent results were obtained, but the calculation of I^{-1} was very slow. For some values of the arguments the IMSL routine MDBETI failed to converge to a solution after 100 iterations of a Newton's method, with the required calculation of I(x) at each step. An inversion routine was written by the author (see MYBETI, Appendix B) which improved the

situation, but finally Eq (21) was seen to be much more convenient, since one integration point cost only one evaluation of I(x) rather than many.

Inverting I(x) was necessary for another purpose (calculating a,b from $e_7=e_8=0$ for given p and q) and this was done using a combination of false position (14:112-117) and Muller's method (14:136 - 145). (See MYBETI in Appendix B.)

Numerical integration required for computing $\underline{\overline{X}}N_{p,q}$ and CVM was done using Gaussian Quadrature (14:401-412 and 13:103). Sixteen point quadrature was first used but an increase to twenty-four point quadrature was made for safety. The fact that $\underline{\overline{X}}N_{p,q}$ changed only in the fourth decimal point in this transition was evidence that sufficient accuracy was being obtained.

Large amounts of computer time was saved by arranging to compute the twenty-four values of I needed in Eq (21) only once each time the true population changed rather than six times for the six estimators multiplied by 500 times for the 500 samples. Even with this and many other economy measures, possibly 90% of the computation time was used in MDBETA and the routines used by MDBETA.

Because some of the estimators were so slow (as much as two CP seconds for one sample on the Cyber 74) the 144 groups of 500 samples had to be broken into many computer runs. Reruns were impossible for format changes and the data was accumulated in a single data file in nearly random order, so small programs were written to sort the data and reformat it to produce the listing in Appendix A and the Tables in Chapter V. Thus, these outputs do not come directly from the routines in Appendix B.

V. Results, Conclusions, and Recommendations

Results

In Appendix A are the outputs from the study, sorted by sample size, population, seed number, and estimator. Means and other statistics are reported for six variables in each case, over the 500 samples involved. These variables are, in order, a, b-1, $p-p_{true}$, $q-q_{true}$, ln(CVM), and the criterion variable ($e_1^2+e_2^2+e_3^2+e_4^2$ in the case of FULLM for instance). The correlation coefficient matrix of these six variables is shown, and finally four other statistics are reported. The first of these is the average number of function evaluations required in the iterative solution scheme. The second is the proportion of samples which required use of FCN2 (ZSPOW applied to two-parameter problems) and third is similar, for FCN4. The last is the proportion of samples for which the method either could not converge (as in MMLE when $p < 1$ or $q < 1$) or was flagged as having possibly diverged because the criterion variable was larger than .01 or because ZSPOW reported a failure. Most of these divergent cases were acceptable, and in any case everything useful was done to minimize them.

Tables I and II on the following pages contain a summary of the mean and the standard error of the mean for the ln (CVM) for the same 144 cases in Appendix A. In Table III are the grand mean and the mean effects of the four variables of the design. These show that, as expected, the two seed numbers had negligible overall differences. The larger sample size gives a smaller CVM by a factor of approximately

$$e^{-2(.3879)} \doteq .46 \qquad (66)$$

TABLE I

SAMPLE SIZE: 10

PTRUE	.5	.5	.5	1.0	1.0	2.0
QTRUE	.5	1.0	2.0	1.0	2.0	2.0
FULLM	-4.4407	-4.4460	-4.2168	-4.6369	-4.5614	-4.8146
	.0456	.0454	.0484	.0466	.0469	.0497
	-4.5201	-4.4021	-4.2542	-4.6355	-4.5837	-4.7480
	.0448	.0466	.0472	.0494	.0499	.0493
MME1	-4.5429	-4.5904	-4.2101	-4.6880	-4.6730	-4.8094
	.0459	.0492	.0459	.0455	.0455	.0480
	-4.6179	-4.5725	-4.2531	-4.7092	-4.6944	-4.7581
	.0448	.0516	.0424	.0487	.0491	.0469
MME2	-4.5235	-4.6183	-4.6468	-4.7049	-4.7680	-4.9533
	.0444	.0422	.0507	.0435	.0460	.0519
	-4.5975	-4.6306	-4.6804	-4.7387	-4.8093	-4.8770
	.0438	.0457	.0507	.0478	.0503	.0519
MME3	-4.7089	-4.6501	-4.3306	-4.8903	-4.7156	-4.9319
	.0494	.0500	.0482	.0516	.0502	.0489
	-4.7937	-4.5715	-4.3593	-4.8093	-4.6684	-4.8591
	.0492	.0487	.0483	.0508	.0499	.0490
MME4	-4.6031	-4.5887	-4.3011	-4.7765	-4.6498	-4.8901
	.0468	.0485	.0472	.0498	.0488	.0512
	-4.6827	-4.5034	-4.3398	-4.7086	-4.6311	-4.7901
	.0461	.0461	.0472	.0505	.0501	.0477
MMLE	-2.5538	-2.9962	-2.6896	-3.4926	-3.7882	-4.1722
	.0492	.0574	.0491	.0695	.0696	.0705
	-2.5732	-2.8213	-2.6754	-3.3118	-3.8082	-3.9503
	.0480	.0556	.0488	.0645	.0684	.0707

TABLE II

SAMPLE SIZE: 20						
PTRUE	.5	.5	.5	1.0	1.0	2.0
QTRUE	.5	1.0	2.0	1.0	2.0	2.0
FULLM	-5.1328	-5.1526	-4.8838	-5.4349	-5.3458	-5.5640
	.0449	.0493	.0489	.0546	.0550	.0536
	-5.2248	-5.1930	-4.9129	-5.4383	-5.3488	-5.4999
	.0456	.0515	.0523	.0532	.0537	.0502
MME1	-5.4728	-5.4101	-5.0910	-5.4968	-5.5454	-5.5681
	.0535	.0521	.0573	.0521	.0529	.0505
	-5.5459	-5.4562	-5.1588	-5.5429	-5.5923	-5.5443
	.0516	.0526	.0586	.0531	.0551	.0486
MME2	-5.4509	-5.4613	-5.4676	-5.5735	-5.5884	-5.6575
	.0510	.0516	.0492	.0533	.0519	.0507
	-5.5241	-5.5259	-5.4615	-5.6194	-5.6374	-5.6456
	.0497	.0524	.0498	.0545	.0553	.0506
MME3	-5.3085	-5.2792	-4.9935	-5.5422	-5.3159	-5.6148
	.0521	.0550	.0544	.0562	.0516	.0525
	-5.3826	-5.2775	-4.9077	-5.5397	-5.3743	-5.6116
	.0523	.0552	.0539	.0551	.0547	.0540
MME4	-5.2814	-5.2419	-5.0137	-5.4518	-5.3062	-5.5339
	.0510	.0541	.0557	.0560	.0534	.0532
	-5.3476	-5.2489	-4.9433	-5.4371	-5.3485	-5.5075
	.0503	.0543	.0549	.0538	.0563	.0548
MMLE	-2.6763	-3.7172	-3.7504	-4.3622	-5.1479	-5.2134
	.0421	.0605	.0597	.0679	.0634	.0618
	-2.6947	-3.6829	-3.6242	-4.2849	-5.1000	-5.1391
	.0450	.0621	.0593	.0661	.0646	.0668

TABLE III

Grand Mean:	-4.7839		
Sample Size Effects:	+0.3879	n=10	
	-0.3879	n=20	
Seed Number Effects:	-0.0025	seed #1	
	+0.0025	seed #2	
Population Effects:	+0.1922	p=0.5	q=0.5
	+0.1157	p=0.5	q=1.0
	+0.3187	p=0.5	q=2.0
	-0.1255	p=1.0	q=1.0
	-0.1745	p=1.0	q=2.0
	-0.3266	p=2.0	q=2.0
Estimator Effects:	-0.1074	FULLM	
	-0.2387	MME1	
	-0.3478	MME2	
	-0.2342	MME3	
	-0.1797	MME4	
	+1.1078	MMLE	

TABLE IV

SAMPLE SIZE: 10						
PTRUE	.5	.5	.5	1.0	1.0	2.0
QTRUE	.5	1.0	2.0	1.0	2.0	2.0
FULLM	-4.4804	-4.4241	-4.2355	-4.6362	-4.5726	-4.7813
	.0320	.0325	.0338	.0340	.0342	.0350
MME1	-4.5804	-4.5815	-4.2316	-4.6986	-4.6837	-4.7838
	.0320	.0356	.0312	.0333	.0334	.0336
MME2	-4.5605	-4.6245	-4.6636	-4.7218	-4.7887	-4.9152
	.0312	.0311	.0358	.0323	.0341	.0367
MME3	-4.7513	-4.6108	-4.3450	-4.8498	-4.6920	-4.8955
	.0349	.0349	.0341	.0362	.0354	.0346
MME4	-4.6429	-4.5461	-4.3205	-4.7426	-4.6405	-4.8401
	.0328	.0335	.0334	.0355	.0349	.0350
MMLE	-2.5635	-2.9088	-2.6825	-3.4022	-3.7982	-4.0613
	.0344	.0400	.0346	.0475	.0487	.0500

TABLE V

SAMPLE SIZE: 20						
PTRUE	.5	.5	.5	1.0	1.0	2.0
QTRUE	.5	1.0	2.0	1.0	2.0	2.0
FULLM	-5.1788	-5.1728	-4.8984	-5.4366	-5.3473	-5.5320
	.0320	.0356	.0358	.0381	.0384	.0367
MME1	-5.5094	-5.4332	-5.1249	-5.5199	-5.5689	-5.5562
	.0372	.0370	.0410	.0372	.0382	.0350
MME2	-5.4875	-5.4936	-5.4646	-5.5965	-5.6129	-5.6516
	.0356	.0368	.0350	.0381	.0379	.0358
MME3	-5.3456	-5.2784	-4.9506	-5.5410	-5.3451	-5.6132
	.0369	.0390	.0383	.0393	.0376	.0376
MME4	-5.3145	-5.2454	-4.9785	-5.4445	-5.3274	-5.5207
	.0358	.0383	.0391	.0388	.0388	.0382
MMLE	-2.6855	-3.7001	-3.6873	-4.3236	-5.1240	-5.1763
	.0308	.0433	.0421	.0474	.0452	.0455

Populations with $p < 1$ or $q < 1$ are apparently more difficult to estimate than others, and the case p=0.5 and q=2.0 is worst of the six. Population p=q=2 is estimated best by these methods. The estimated treatment effects show that MMLE performed poorly while MME2 seems to have performed best.

Tables IV and V show the same data as Tables I and II, but with the two series of 500 samples collapsed into one collection of 1000 data points. A pessimistic estimate of standard error is .05, so differences in estimated effects much larger than that can be considered significant. Thus, MMLE is significantly poorer than the other estimates and MME2 is probably better than the others, while the others may be about equally good. This significance test is not formalized because none of the usual assumptions are likely to be met (normality, independence, equal within-cell variances).

Conclusions and Recommendations

MME2 seems to be clearly better than the other estimates over this range of populations and sample sizes, if CVM is an appropriate measure. Each estimator is optimal with respect to a defensible measure of its own, however, since the minimization of $e_1^2+e_2^2+e_3^2+e_4^2$ (for instance) is just as intuitively attractive and meaningful as minimization of CVM. Therefore, such results must be used with care, and the behavior of the mean square errors should augment that of CVM in a complete evaluation of these estimators.

MME2 is apparently the best of these estimators in the sense agreed upon, but recorded CP times show that it is also the slowest of them all

(at least with the implementation used here). FULLM is by far the fastest estimator used here, followed by MMLE, MME3, MME4, MME1, and MME2 in order from fastest to slowest. FULLM is over 100 times faster than MME2, and if that extra cost is significant to a user it should be considered. For a single sample, even MME2 can be computed in less than 3 CP seconds, however, so the difference is really important only for Monte Carlo analysis such as this.

Future work in this area could center on using such things as mode statistics in place of the first moment or using an extension of the MME2 estimator to use the first two and last two order statistics along with the first two moments. A very promising estimator is given by $e_1=e_2=0$ together with:

$$E\left(\frac{\underline{\overline{X}}(1) + \underline{\overline{X}}(2)}{2}\right) = \frac{x(1) + x(2)}{2} \tag{67}$$

$$E\left(\frac{\underline{\overline{X}}(n-1) + \underline{\overline{X}}(n)}{2}\right) = \frac{x(n-1) + x(n)}{2} \tag{68}$$

Since the statistics involved would have smaller variance than $\underline{\overline{X}}(1)$ or $\underline{\overline{X}}(2)$ alone. The superior performance of MME2 is probably due to the small variance of $\underline{\overline{X}}(1)$ and $\underline{\overline{X}}(n)$, and if that can be taken advantage of, then the location parameters can be better estimated. Much work in more theoretical areas needs to be done in order to answer questions like the appropriateness of one distance measure over another.

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VITA

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APPENDIX A

COMPUTER OUTPUT

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0358	.0144	.0051	.1146
.0251	.0109	.0045	.1013
.1366	.7400	.0380	.8493
.1188	.4046	.0279	.6249
-4.4407	20.7584	.0456	1.0191
.0046	.0029	.0024	.0539

CORRELATION COEFFICIENTS:

1.000					
-.093	1.000				
-.776	.286	1.000			
-.409	.621	.711	1.000		
.016	-.058	-.060	-.060	1.000	
.033	-.027	-.046	-.047	.042	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.9260	0.0000	.4640	.0460

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0303	.0507	.0100	.2231
.0253	.0598	.0109	.2431
.0743	.4807	.0308	.6893
.0631	.3674	.0270	.6028
-4.5429	21.6874	.0458	1.0245
.0014	.0001	.0005	.0116

CORRELATION COEFFICIENTS:

1.000					
-.012	1.000				
-.850	.080	1.000			
-.143	.796	.379	1.000		
-.006	.005	-.088	-.082	1.000	
-.030	.224	.013	.057	.172	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
24.4840	.3560	.3380	.0480

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.2329	.4406	.0278	.6216
.1616	.2455	.0209	.4684
1.0641	8.7337	.1233	2.7571
.8038	5.2958	.0964	2.1563
-4.5235	21.4439	.0443	.9910
.0003	.0000	.0001	.0014

CORRELATION COEFFICIENTS:

1.000					
.008	1.000				
-.914	.095	1.000			
-.137	.886	.331	1.000		
-.205	.219	.144	.171	1.000	
-.491	-.010	.409	.034	.161	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
15.3660	.1920	.1520	.0040

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0394	.0291	.0074	.1660
.0351	.0250	.0069	.1542
.1366	.7400	.0380	.8493
.1188	.4046	.0279	.6249
-4.7089	23.3913	.0493	1.1033
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.173	1.000				
-.866	.292	1.000			
-.483	.687	.711	1.000		
.092	-.025	-.163	-.165	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

```
SAMPLE SIZE:    10   ESTIMATOR: MMF4    SEED #:    1
    TRUE P:     .500   TRUE Q:     .500

              MEAN   MEAN SQUARE      STD ERROR       STD DEV
           -.0426          .0216          .0063         .1406
            .0384          .0179          .0057         .1282
            .1366          .7400          .0380         .8493
            .1188          .4046          .0279         .6249
          -4.6031        22.2797          .0467        1.0444
           0.0000         0.0000         0.0000        0.0000

CORRELATION COEFFICIENTS:
 1.000
 -.128  1.000
 -.844   .268  1.000
 -.452   .667   .711  1.000
  .052  -.009  -.126  -.131  1.000
                                    1.000

ITERATIONS  FCN2 USED  FCN4 USED  DIVERGED
    0.0000     0.0000     0.0000    0.0000


SAMPLE SIZE:    10   ESTIMATOR: MMLE    SEED #:    1
    TRUE P:     .500   TRUE Q:     .500

              MEAN   MEAN SQUARE      STD ERROR       STD DEV
           -.3815        13.7488          .1649        3.6883
           -.5631         1.0695          .0388         .8674
            .1366          .7400          .0380         .8493
            .1188          .4046          .0279         .6249
          -2.5538         7.7316          .0492        1.0998
    9391.5850715181189.8925          1119.8029    25039.5551

CORRELATION COEFFICIENTS:
 1.000
 -.124  1.000
  .011   .213  1.000
  .026   .323   .711  1.000
 -.113  -.378  -.484  -.623  1.000
 -.322   .108   .051   .095  -.167  1.000

ITERATIONS  FCN2 USED  FCN4 USED  DIVERGED
    2.1980      .0820     0.0000     .9200
```

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0359	.0227	.0065	.1462
.0449	.0561	.0104	.2326
.2935	9.8500	.1397	3.1247
.4476	30.3813	.2457	5.4937
-4.5201	21.4337	.0448	1.0012
.0101	.0091	.0042	.0949

CORRELATION COEFFICIENTS:

1.000					
-.631	1.000				
-.780	.882	1.000			
-.707	.911	.986	1.000		
.011	009	-.004	.007	1.000	
-.068	33	.055	.029	.068	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.5140	0.0000	.4200	.0420

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0171	.0210	.0064	.1438
.0257	.0152	.0054	.1206
.0574	.2917	.0240	.5370
.0856	.4143	.0285	.6380
-4.6179	22.3250	.0447	.9998
.0010	.0000	.0002	.0051

CORRELATION COEFFICIENTS:

1.000					
-.040	1.000				
-.700	.351	1.000			
-.162	.825	.605	1.000		
-.013	-.021	-.082	-.069	1.000	
-.041	.295	.096	.146	.136	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
22.8880	.3260	.3060	.0420

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1716	.2939	.0230	.5142
.1386	.1774	.0178	.3977
.8523	6.4866	.1073	2.4000
.7388	4.8735	.0930	2.0803
-4.5975	22.0943	.0438	.9784
.0003	.0000	.0001	.0016

CORRELATION COEFFICIENTS:

1.000					
-.025	1.000				
-.882	.163	1.000			
-.179	.873	.440	1.000		
-.194	.181	.152	.145	1.000	
-.500	.002	.466	.103	.129	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
14.2840	.1560	.1340	.0080

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0405	.0448	.0093	.2078
.0591	.1161	.0150	.3356
.2935	9.8500	.1397	3.1247
.4476	30.3813	.2457	5.4937
-4.7937	24.1866	.0491	1.0986
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.685	1.000				
-.819	.879	1.000			
-.740	.907	.986	1.000		
.088	-.060	-.070	-.052	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0434	.0327	.0079	.1757
.0601	.0836	.0127	.2829
.2935	9.8500	.1397	3.1247
.4476	30.3813	.2457	5.4937
-4.6827	22.9857	.0460	1.0288
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.658	1.000				
-.807	.877	1.000			
-.728	.908	.986	1.000		
.044	-.020	-.040	-.023	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1558	2.6616	.0726	1.6240
-.5064	.7296	.0308	.6879
.2935	9.8500	.1397	3.1247
.4476	30.3813	.2457	5.4937
-2.5732	7.7716	.0480	1.0725
8185.3309563529110.7419		996.5234	22282.9412

CORRELATION COEFFICIENTS:

1.000					
-.263	1.000				
-.052	.375	1.000			
-.047	.372	.986	1.000		
-.082	-.450	-.200	-.156	1.000	
-.200	.096	-.010	-.008	-.216	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.3520	.0960	0.0000	.9020

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0265	.0091	.0041	.0917
-.0410	.0670	.0114	.2555
.1209	.9170	.0425	.9499
.1478	5.8488	.1080	2.4139
-4.4460	20.7969	.0454	1.0147
.0151	.0690	.0117	.2622

CORRELATION COEFFICIENTS:

1.000					
-.211	1.000				
-.825	.457	1.000			
-.386	.682	.728	1.000		
-.008	-.229	-.056	-.085	1.000	
-.029	.007	.013	-.008	.051	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.5460	0.0000	.4660	.0740

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0040	.0026	.0023	.0508
-.0473	.0521	.0100	.2234
-.0375	.1512	.0173	.3871
-.1441	.5507	.0326	.7279
-4.5904	22.2803	.0492	1.0992
.0019	.0000	.0002	.0042

CORRELATION COEFFICIENTS:

1.000					
-.042	1.000				
-.710	.316	1.000			
-.248	.667	.669	1.000		
-.021	-.365	-.168	-.350	1.000	
-.017	.128	-.014	.092	.223	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
38.0500	.5740	.5560	.0880

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0677	.0506	.0096	.2145
.4049	.8394	.0368	.8219
.6246	3.5560	.0796	1.7793
2.0520	16.4128	.1562	3.4931
-4.6183	22.2167	.0421	.9424
.0002	.0000	.0000	.0010

CORRELATION COEFFICIENTS:

1.000					
-.006	1.000				
-.912	.111	1.000			
-.223	.860	.413	1.000		
-.170	-.315	.117	-.221	1.000	
-.318	.119	.315	.144	.080	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
24.1280	.3600	.3280	.0040

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0226	.0123	.0049	.1085
.0328	.1300	.0161	.3591
.1209	.9170	.0425	.9499
.1478	5.8488	.1080	2.4139
-4.6501	22.8696	.0499	1.1164
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.269	1.000				
-.896	.445	1.000			
-.473	.704	.728	1.000		
-.048	-.200	-.031	-.100	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMF4 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0234	.0090	.0041	.0921
.0169	.0942	.0137	.3065
.1209	.9170	.0425	.9499
.1478	5.8488	.1080	2.4139
-4.5887	22.2300	.0485	1.0835
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.236	1.000				
-.881	.424	1.000			
-.454	.684	.728	1.000		
-.047	-.209	-.026	-.093	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1534	2.7897	.0744	1.6632
-.3616	.3578	.0213	.4764
.1209	.9170	.0425	.9499
.1478	5.8488	.1080	2.4139
-2.9962	10.6218	.0574	1.2825
10808.5825	755223439.3249	1129.9540	25266.5388

CORRELATION COEFFICIENTS:

1.000					
-.082	1.000				
.009	.426	1.000			
.011	.530	.728	1.000		
-.130	-.708	-.308	-.296	1.000	
-.129	.190	-.026	.015	-.221	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.6560	.1180	0.0000	.8940

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0233	.0057	.0032	.0718
-.0523	.0579	.0105	.2348
.0617	.4298	.0292	.6527
-.0309	1.5318	.0553	1.2373
-4.4021	20.4614	.0465	1.0407
.0024	.0000	.0003	.0065

CORRELATION COEFFICIENTS:

1.000					
-.164	1.000				
-.751	.360	1.000			
-.398	.688	.732	1.000		
-.012	-.139	-.041	-.130	1.000	
.143	.125	-.170	-.028	.212	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.1020	0.0000	.4320	.0720

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0035	.0034	.0026	.0578
-.0544	.0527	.0100	.2230
-.0532	.1568	.0176	.3924
-.2052	.6704	.0354	.7926
-4.5725	22.2380	.0516	1.1532
.0020	.0000	.0002	.0055

CORRELATION COEFFICIENTS:

1.000					
-.108	1.000				
-.714	.327	1.000			
-.264	.701	.632	1.000		
-.043	-.260	-.101	-.282	1.000	
-.035	.208	-.053	.081	.237	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
35.3980	.5420	.5220	.0880

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0674	.0568	.0102	.2285
.3383	.7118	.0346	.7729
.5708	3.4945	.0796	1.7801
1.6801	13.7306	.1477	3.3027
-4.6306	22.4838	.0456	1.0203
.0001	.0000	.0000	.0005

CORRELATION COEFFICIENTS:

1.000					
-.023	1.000				
-.937	.113	1.000			
-.217	.848	.383	1.000		
-.194	-.254	.161	-.172	1.000	
-.522	.045	.459	.080	.046	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
21.9960	.3200	.2820	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0190	.0088	.0041	.0919
.0106	.1135	.0151	.3367
.0617	.4298	.0292	.6527
-.0309	1.5318	.0553	1.2373
-4.5715	22.0827	.0487	1.0880
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.261	1.000				
-.867	.366	1.000			
-.527	.754	.732	1.000		
-.073	-.117	-.002	-.136	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMF4 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0201	.0067	.0035	.0791
.0014	.0832	.0129	.2884
.0617	.4298	.0292	.6527
-.0309	1.5318	.0553	1.2373
-4.5034	21.3428	.0461	1.0305
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.225	1.000				
-.846	.340	1.000			
-.498	.719	.732	1.000		
-.060	-.119	-.014	-.156	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1650	6.8426	.1168	2.6106
-.4282	.5900	.0285	.6376
.0617	.4298	.0292	.6527
-.0309	1.5318	.0553	1.2373
-2.8213	9.5026	.0555	1.2421
12072.1791	900614681.4612	1228.7206	27475.0282

CORRELATION COEFFICIENTS:

1.000					
-.289	1.000				
.018	.347	1.000			
.022	.472	.732	1.000		
-.088	-.515	-.403	-.473	1.000	
-.179	.143	-.018	.024	-.235	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.5080	.1120	0.0000	.8840

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0108	.0046	.0030	.0673
-.2753	.1415	.0115	.2563
-.0568	.3140	.0249	.5575
-.9071	2.0321	.0492	1.0997
-4.2168	18.9521	.0484	1.0818
.0779	2.3965	.0691	1.5461

CORRELATION COEFFICIENTS:

1.000					
-.199	1.000				
-.827	.195	1.000			
-.459	.404	.766	1.000		
-.030	-.108	-.109	-.169	1.000	
-.907	.186	.694	.360	.015	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.9820	0.0000	.5260	.1360

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0024	.0003	.0008	.0171
-.3009	.1466	.0106	.2367
-.1965	.1028	.0113	.2534
-1.2958	1.9337	.0226	.5047
-4.2101	18.7753	.0458	1.0248
.0047	.0002	.0006	.0131

CORRELATION COEFFICIENTS:

1.000					
-.014	1.000				
-.595	.048	1.000			
-.409	.290	.805	1.000		
-.043	-.054	-.258	-.279	1.000	
.025	.384	-.198	-.053	.280	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
44.3620	.7360	.7060	.1500

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0200	.0055	.0032	.0712
.1777	.5461	.0321	.7173
.2844	1.2544	.0484	1.0833
1.5123	13.7818	.1516	3.3904
-4.6468	22.8757	.0507	1.1326
.0001	.0000	.0000	.0003

CORRELATION COEFFICIENTS:

1.000					
-.033	1.000				
-.915	.098	1.000			
-.283	.729	.477	1.000		
-.161	-.431	.102	-.254	1.000	
-.189	.031	.148	.051	.011	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
34.5680	.6160	.5240	.0060

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0058	.0025	.0022	.0492
-.1732	.1514	.0156	.3484
-.0568	.3140	.0249	.5575
-.9071	2.0321	.0492	1.0997
-4.3306	19.9115	.0481	1.0759
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.175	1.000				
-.913	.154	1.000			
-.619	.446	.766	1.000		
-.127	.036	.001	-.095	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0067	.0018	.0019	.0421
-.2000	.1342	.0137	.3069
-.0568	.3140	.0249	.5575
-.9071	2.0321	.0492	1.0997
-4.3011	19.6090	.0471	1.0535
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.152	1.000				
-.908	.135	1.000			
-.600	.404	.766	1.000		
-.119	.003	-.018	-.118	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1377	3.5449	.0840	1.8777
-.4891	.3596	.0155	.3470
-.0568	.3140	.0249	.5575
-.9071	2.0321	.0492	1.0997
-2.6896	8.4369	.0491	1.0969
16919.2884		1428.5447	31943.2304

CORRELATION COEFFICIENTS:

1.000					
-.008	1.000				
.016	.338	1.000			
.034	.577	.766	1.000		
-.111	-.506	-.429	-.501	1.000	
-.082	.210	.034	.073	-.209	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.0360	.0720	0.0000	.9340

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0075	.0010	.0014	.0313
-.2550	.1522	.0132	.2953
-.0786	.2418	.0217	.4854
-.8485	8.1091	.1216	2.7183
-4.2542	19.2098	.0472	1.0545
.0177	.0834	.0129	.2883

CORRELATION COEFFICIENTS:

1.000					
-.152	1.000				
-.688	.404	1.000			
-.358	.589	.795	1.000		
-.057	-.055	-.100	-.031	1.000	
.021	.002	-.034	-.018	.040	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.4020	0.0000	.5400	.1420

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0035	.0003	.0007	.0158
-.2836	.1385	.0108	.2409
-.2060	.0876	.0095	.2126
-1.3046	1.9377	.0217	.4856
-4.2531	18.9861	.0424	.9472
.0044	.0001	.0004	.0089

CORRELATION COEFFICIENTS:

1.000					
.020	1.000				
-.478	.096	1.000			
-.216	.422	.740	1.000		
-.041	.003	-.310	-.305	1.000	
.046	.372	-.250	-.040	.317	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
42.1620	.7080	.6680	.1520

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0171	.0037	.0026	.0580
.1624	.4942	.0306	.6840
.2101	.7559	.0377	.8436
1.2870	12.8121	.1494	3.3400
-4.6804	23.1867	.0506	1.1315
.0001	.0000	.0000	.0006

CORRELATION COEFFICIENTS:

1.000					
-.112	1.000				
-.920	.219	1.000			
-.332	.760	.547	1.000		
-.181	-.381	.091	-.239	1.000	
-.024	.198	.021	.105	-.040	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
32.4320	.5780	.5120	.0020

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0034	.0012	.0015	.0341
-.1470	.1927	.0185	.4136
-.0786	.2418	.0217	.4854
-.8485	8.1091	.1216	2.7183
-4.3593	20.1687	.0483	1.0794
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.290	1.000				
-.843	.395	1.000			
-.558	.604	.795	1.000		
-.137	.089	-.012	-.001	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 2

TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0047	.0009	.0013	.0297
-.1743	.1601	.0161	.3602
-.0786	.2418	.0217	.4854
-.8485	8.1091	.1216	2.7183
-4.3398	19.9469	.0472	1.0551
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.243	1.000				
-.824	.374	1.000			
-.514	.584	.795	1.000		
-.117	.060	-.035	-.010	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 2

TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1422	3.4146	.0824	1.8424
-.4679	.3700	.0174	.3886
-.0786	.2418	.0217	.4854
-.8485	8.1091	.1216	2.7183
-2.6754	8.3440	.0487	1.0890
13589.0473		1280.7684	28638.8526

CORRELATION COEFFICIENTS:

1.000					
-.038	1.000				
.049	.518	1.000			
.013	.560	.795	1.000		
-.108	-.496	-.403	-.205	1.000	
-.112	.256	.075	.033	-.309	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
1.8820	.0620	0.0000	.9460

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0285	.0407	.0089	.1998
.0118	.0526	.0102	.2291
.2941	4.0266	.0888	1.9850
.2944	5.5833	.1048	2.3445
-4.6369	22.5848	.0466	1.0410
.0032	.0012	.0016	.0351

CORRELATION COEFFICIENTS:

1.000					
-.187	1.000				
-.790	.428	1.000			
-.387	.835	.695	1.000		
.090	-.062	-.071	-.038	1.000	
-.159	.109	.183	.156	.039	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.6480	0.0000	.3520	.0460

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0142	.0313	.0079	.1765
.0016	.0304	.0078	.1743
.0076	1.0393	.0456	1.0194
-.0189	1.0187	.0451	1.0091
-4.6880	23.0093	.0454	1.0158
.0017	.0000	.0001	.0033

CORRELATION COEFFICIENTS:

1.000					
-.019	1.000				
-.782	.233	1.000			
-.263	.774	.583	1.000		
.161	-.093	-.143	-.080	1.000	
-.273	.205	.295	.263	-.026	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
41.7400	.5780	.5620	.1220

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.3276	.4726	.0270	.6044
.1805	.2346	.0201	.4495
2.2081	18.3656	.1643	3.6728
1.5755	11.8095	.1366	3.0541
-4.7049	23.0780	.0434	.9707
.0007	.0000	.0001	.0023

CORRELATION COEFFICIENTS:

1.000					
.029	1.000				
-.907	.124	1.000			
-.136	.868	.387	1.000		
-.011	-.000	-.050	-.036	1.000	
-.566	-.046	.477	.032	.052	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
26.8580	.3680	.2980	.0160

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0665	.0855	.0127	.2848
.0542	.1095	.0146	.3264
.2941	4.0266	.0888	1.9850
.2944	5.5833	.1048	2.3445
-4.8903	25.2459	.0516	1.1536
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.261	1.000				
-.846	.460	1.000			
-.428	.865	.695	1.000		
.234	-.185	-.238	-.188	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 1

TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0531	.0607	.0108	.2406
.0420	.0783	.0124	.2767
.2941	4.0266	.0888	1.9850
.2944	5.5833	.1048	2.3445
-4.7765	24.0544	.0498	1.1135
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.228	1.000				
-.830	.444	1.000			
-.411	.853	.695	1.000		
.169	-.134	-.179	-.133	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 1

TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0169	.3891	.0279	.6236
-.3047	.3663	.0234	.5230
.2941	4.0266	.0888	1.9850
.2944	5.5833	.1048	2.3445
-3.4926	14.6062	.0694	1.5517
12219.8461	961146071.7260	1274.2225	28492.4803

CORRELATION COEFFICIENTS:

1.000					
-.450	1.000				
-.235	.384	1.000			
-.111	.548	.695	1.000		
.051	-.580	-.305	-.258	1.000	
.046	.080	-.079	-.063	-.158	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.5100	.2680	0.0000	.7520

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0123	.0416	.0091	.2037
.2361	27.3347	.2336	5.2229
.3105	24.6891	.2218	4.9591
9.2784	42106.4649	9.1674	204.9887
-4.6355	22.7079	.0494	1.1046
.0045	.0017	.0018	.0409

CORRELATION COEFFICIENTS:

1.000					
-.502	1.000				
-.664	.962	1.000			
-.496	1.000	.959	1.000		
.113	-.047	-.075	-.044	1.000	
-.282	.006	.108	-.002	-.028	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.5760	0.0000	.4080	.0520

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0014	.0328	.0081	.1810
-.0091	.0218	.0066	.1475
-.0743	.8769	.0417	.9335
-.1370	.5548	.0327	.7321
-4.7092	23.3587	.0486	1.0872
.0015	.0000	.0002	.0035

CORRELATION COEFFICIENTS:

1.000					
-.049	1.000				
-.827	.203	1.000			
-.290	.754	.530	1.000		
.123	-.144	-.136	-.216	1.000	
-.230	.222	.258	.263	-.058	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
38.1960	.5500	.5200	.0900

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.2820	.4674	.0279	.6228
.1614	.2481	.0211	.4712
1.7657	15.0481	.1545	3.4540
1.2004	9.0795	.1236	2.7638
-4.7387	23.5959	.0478	1.0678
.0006	.0000	.0001	.0021

CORRELATION COEFFICIENTS:

1.000					
.035	1.000				
-.902	.088	1.000			
-.133	.852	.368	1.000		
-.093	.062	.022	-.013	1.000	
-.614	-.010	.548	.076	.050	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
23.3900	.3100	.2500	.0120

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0430	.0827	.0127	.2843
.3563	51.2372	.3197	7.1491
.3105	24.6891	.2218	4.9591
9.2784	42106.4649	9.1674	204.9887
-4.8093	24.4191	.0508	1.1356
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.521	1.000				
-.696	.962	1.000			
-.514	1.000	.959	1.000		
.248	-.068	-.145	-.060	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.4658	96.3329	.4384	9.8039
21.0411	221201.1190	21.0123	469.8493
.3105	24.6891	.2218	4.9591
9.2784	42106.4649	9.1674	204.9887
-4.7086	23.4457	.0505	1.1289
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-1.000	1.000				
-.962	.957	1.000			
-1.000	1.000	.959	1.000		
-.121	.125	.049	.123	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1536	6.2500	.1116	2.4953
4.0247	9525.7312	4.3611	97.5168
.3105	24.6891	.2218	4.9591
9.2784	42106.4649	9.1674	204.9887
-3.3118	13.0437	.0644	1.4408
12858.3660	998854882.3855	1291.1369	28870.6998

CORRELATION COEFFICIENTS:

1.000					
-.896	1.000				
-.866	.958	1.000			
-.896	1.000	.959	1.000		
-.059	.018	-.098	.017	1.000	
-.030	-.020	-.042	-.021	-.124	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.6940	.2140	0.0000	.7920

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0025	.0212	.0065	.1456
-.1057	.0915	.0127	.2834
.0579	5.6209	.1060	2.3701
-.3593	6.1679	.1099	2.4574
-4.5614	21.9016	.0468	1.0466
.0046	.0008	.0013	.0285

CORRELATION COEFFICIENTS:

1.000					
-.121	1.000				
-.897	.222	1.000			
-.465	.689	.612	1.000		
-.031	-.121	.031	-.074	1.000	
.058	.306	-.017	.141	.120	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.9020	0.0000	.4080	.0980

SAMPLE SIZE: 10 ESTIMATOR: MME1 SFED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0157	.0069	.0036	.0813
-.1192	.0759	.0111	.2484
-.2778	.6277	.0332	.7419
-.7857	2.0093	.0528	1.1798
-4.6730	22.8701	.0455	1.0165
.0022	.0000	.0002	.0044

CORRELATION COEFFICIENTS:

1.000					
-.071	1.000				
-.745	.308	1.000			
-.409	.710	.741	1.000		
.052	-.267	-.104	-.257	1.000	
-.005	.267	.036	.155	.102	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
47.8180	.6780	.6460	.1200

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1041	.0746	.0113	.2525
.2414	.5044	.0299	.6679
1.1333	8.0455	.1163	2.6002
1.9984	17.4847	.1643	3.6730
-4.7680	23.7887	.0459	1.0272
.0003	.0000	.0001	.0012

CORRELATION COEFFICIENTS:

1.000					
.063	1.000				
-.915	.008	1.000			
-.220	.794	.402	1.000		
-.086	-.259	.055	-.235	1.000	
-.370	.062	.330	.093	.126	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
33.0000	.4780	.4200	.0060

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0194	.0431	.0092	.2066
-.0160	.1619	.0180	.4020
.0579	5.6209	.1060	2.3701
-.3593	6.1679	.1099	2.4574
-4.7156	23.4931	.0501	1.1207
0.0000	0.0000	0.0000	0.0000

CORRELATION COFFFICIENTS:

1.000					
-.198	1.000				
-.928	.242	1.000			
-.522	.722	.612	1.000		
.004	-.139	.000	-.148	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0149	.0313	.0079	.1762
-.0462	.1202	.0154	.3436
.0579	5.6209	.1060	2.3701
-.3593	6.1679	.1099	2.4574
-4.6498	22.8067	.0487	1.0890
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.171	1.000				
-.919	.229	1.000			
-.507	.704	.612	1.000		
-.010	-.150	.014	-.148	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1359	4.1018	.0904	2.0207
-.2854	.3507	.0232	.5189
.0579	5.6209	.1060	2.3701
-.3593	6.1679	.1099	2.4574
-3.7882	16.7648	.0695	1.5537
13880.4572		1315.1785	29408.2845

CORRELATION COEFFICIENTS:

1.000					
-.084	1.000				
-.036	.202	1.000			
.007	.525	.612	1.000		
-.142	-.512	-.098	-.269	1.000	
-.132	.053	-.068	-.058	-.093	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.1300	.2420	0.0000	.7660

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0011	.0089	.0042	.0943
-.1225	.1299	.0152	.3390
-.0861	1.4362	.0535	1.1953
-.3569	10.3939	.1433	3.2042
-4.5837	22.2510	.0498	1.1139
.0036	.0002	.0006	.0131

CORRELATION COEFFICIENTS:

1.000					
-.194	1.000				
-.770	.413	1.000			
-.337	.833	.627	1.000		
.014	-.164	-.070	-.115	1.000	
-.108	.111	.097	.053	.181	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.8080	0.0000	.3940	.1020

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0154	.0050	.0031	.0692
-.1392	.0804	.0110	.2470
-.3297	.3403	.0215	.4812
-.8861	1.8147	.0454	1.0146
-4.6944	23.2386	.0490	1.0959
.0019	.0000	.0001	.0033

CORRELATION COEFFICIENTS:

1.000					
-.051	1.000				
-.667	.295	1.000			
-.242	.753	.610	1.000		
.025	-.317	-.226	-.376	1.000	
-.010	.300	.057	.236	.085	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
46.4500	.6820	.6580	.1020

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0975	.0766	.0116	.2591
.2616	.5439	.0308	.6895
.9687	6.8794	.1090	2.4374
2.0303	18.3791	.1689	3.7759
-4.8093	24.3932	.0503	1.1241
.0002	.0000	.0000	.0007

CORRELATION COEFFICIENTS:

1.000					
.053	1.000				
-.912	.059	1.000			
-.174	.830	.388	1.000		
-.119	-.223	.049	-.214	1.000	
-.503	.039	.490	.163	.060	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
31.2020	.4600	.3800	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0129	.0162	.0057	.1268
-.0379	.2411	.0219	.4896
-.0861	1.4362	.0535	1.1953
-.3569	10.3939	.1433	3.2042
-4.6684	23.0347	.0498	1.1139
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.282	1.000				
-.864	.413	1.000			
-.435	.855	.627	1.000		
.061	-.212	-.128	-.225	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0099	.0118	.0048	.1081
-.0647	.1800	.0188	.4193
-.0861	1.4362	.0535	1.1953
-.3569	10.3939	.1433	3.2042
-4.6311	22.7014	.0501	1.1198
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.250	1.000				
-.833	.401	1.000			
-.407	.842	.627	1.000		
.040	-.228	-.108	-.208	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SFED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0193	1.2461	.0499	1.1161
-.3292	.4775	.0272	.6076
-.0861	1.4362	.0535	1.1953
-.3569	10.3939	.1433	3.2042
-3.8082	16.8361	.0683	1.5276
15565.6165		1408.9582	31505.2626

CORRELATION COEFFICIENTS:

1.000					
-.282	1.000				
-.047	.348	1.000			
-.022	.569	.627	1.000		
-.057	-.490	-.233	-.249	1.000	
-.139	.089	-.084	-.065	-.179	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.5180	.1980	0.0000	.8060

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0273	.0976	.0139	.3112
-.0219	.1095	.0148	.3302
-.0597	17.9526	.1895	4.2366
.0958	26.0206	.2281	5.1001
-4.8146	24.4110	.0496	1.1093
.0187	.0288	.0075	.1686

CORRELATION COEFFICIENTS:

1.000					
-.252	1.000				
-.866	.393	1.000			
-.325	.886	.491	1.000		
.123	-.060	-.147	-.045	1.000	
-.197	.225	.166	.149	-.036	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.9920	0.0000	.2960	.0700

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0650	.0426	.0088	.1959
-.0674	.0373	.0081	.1809
-.8521	1.8539	.0475	1.0620
-.8594	1.6546	.0428	.9571
-4.8094	24.2804	.0480	1.0726
.0022	.0000	.0002	.0036

CORRELATION COEFFICIENTS:

1.000					
-.008	1.000				
-.838	.139	1.000			
-.187	.780	.427	1.000		
.071	-.047	-.167	-.121	1.000	
-.285	.234	.299	.250	.076	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
52.3840	.7580	.7400	.1380

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.2560	.3771	.0250	.5582
.1239	.1942	.0189	.4229
1.9853	17.8292	.1667	3.7267
1.3320	12.0125	.1431	3.1997
-4.9533	25.8790	.0518	1.1591
.0009	.0000	.0001	.0027

CORRELATION COEFFICIENTS:

1.000					
.111	1.000				
-.870	.031	1.000			
-.019	.844	.306	1.000		
.012	-.001	-.050	-.026	1.000	
-.593	-.055	.457	-.019	.058	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
35.7540	.4980	.4160	.0240

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0252	.1808	.0190	.4245
.0417	.2101	.0204	.4565
-.0597	17.9526	.1895	4.2366
.0958	26.0206	.2281	5.1001
-4.9319	25.5151	.0488	1.0914
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.270	1.000				
-.888	.394	1.000			
-.340	.893	.491	1.000		
.138	-.135	-.168	-.133	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MME4 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0020	.1319	.0162	.3632
.0162	.1525	.0174	.3902
-.0597	17.9526	.1895	4.2366
.0958	26.0206	.2281	5.1001
-4.8901	25.2221	.0512	1.1440
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.252	1.000				
-.883	.385	1.000			
-.332	.888	.491	1.000		
.172	-.134	-.211	-.131	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 10 ESTIMATOR: MMLE SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0742	.2271	.0211	.4708
-.1945	.2720	.0216	.4840
-.0597	17.9526	.1895	4.2366
.0958	26.0206	.2281	5.1001
-4.1722	19.8856	.0704	1.5743
11367.1590	875595752.7988	1221.7884	27320.0192

CORRELATION COEFFICIENTS:

1.000					
-.428	1.000				
-.538	.327	1.000			
-.199	.653	.491	1.000		
.095	-.452	-.209	-.139	1.000	
.005	-.025	-.083	-.089	-.063	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.8800	.3940	0.0000	.6180

SAMPLE SIZE: 10 ESTIMATOR: FULLM SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0477	.0801	.0125	.2789
-.0676	.0472	.0092	.2064
-.3186	7.4579	.1213	2.7123
-.4457	3.6577	.0832	1.8598
-4.7480	23.7556	.0492	1.1010
.0094	.0131	.0051	.1142

CORRELATION COEFFICIENTS:

1.000					
-.180	1.000				
-.885	.307	1.000			
-.390	.766	.601	1.000		
.158	-.199	-.169	-.173	1.000	
-.134	.322	.135	.317	-.014	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.9080	0.0000	.3060	.0480

SAMPLE SIZE: 10 ESTIMATOR: MME1 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0840	.0334	.0073	.1623
-.0828	.0362	.0077	.1712
-.9359	1.6829	.0402	.8984
-.9210	1.6422	.0399	.8911
-4.7581	23.7389	.0469	1.0484
.0020	.0000	.0001	.0031

CORRELATION COEFFICIENTS:

1.000					
-.033	1.000				
-.737	.229	1.000			
-.219	.729	.565	1.000		
.112	-.190	-.223	-.267	1.000	
-.230	.223	.247	.248	-.011	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
49.5440	.7280	.6980	.1400

SAMPLE SIZE: 10 ESTIMATOR: MME2 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.2465	.3484	.0240	.5363
.0907	.1639	.0176	.3945
2.0363	18.7035	.1706	3.8154
1.1697	11.3763	.1415	3.1636
-4.8770	25.1297	.0519	1.1596
.0007	.0000	.0001	.0018

CORRELATION COEFFICIENTS:

1.000					
.073	1.000				
-.863	.081	1.000			
-.081	.846	.374	1.000		
.017	-.145	-.075	-.178	1.000	
-.519	-.012	.413	.037	-.005	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
34.1620	.4820	.4120	.0200

SAMPLE SIZE: 10 ESTIMATOR: MME3 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0018	.1481	.0172	.3848
-.0191	.0731	.0121	.2697
-.3186	7.4579	.1213	2.7123
-.4457	3.6577	.0832	1.8598
-4.8591	24.8113	.0490	1.0956
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.219	1.000				
-.917	.344	1.000			
-.402	.806	.601	1.000		
.182	-.251	-.228	-.302	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

```
SAMPLE SIZE:    10   ESTIMATOR: MME4    SEED #:     2
    TRUE P:    2.000    TRUE Q:    2.000

              MEAN   MEAN SQUARE      STD ERROR        STD DEV
             .0177         .1091          .0148          .3299
            -.0360         .0555          .0104          .2327
            -.3186        7.4579          .1213         2.7123
            -.4457        3.6577          .0832         1.8598
           -4.7901       24.0827          .0477         1.0666
            0.0000        0.0000         0.0000         0.0000

CORRELATION COEFFICIENTS:
 1.000
 -.191  1.000
 -.904   .323  1.000
 -.387   .777   .601  1.000
  .156  -.218  -.196  -.253  1.000
                                    1.000

ITERATIONS  FCN2 USED  FCN4 USED   DIVERGED
    0.0000     0.0000     0.0000     0.0000
```

```
SAMPLE SIZE:    10   ESTIMATOR: MMLE    SEED #:     2
    TRUE P:    2.000    TRUE Q:    2.000

              MEAN   MEAN SQUARE      STD ERROR        STD DEV
             .1164         .3386          .0255          .5701
            -.2688         .3390          .0231          .5165
            -.3186        7.4579          .1213         2.7123
            -.4457        3.6577          .0832         1.8598
           -3.9503       18.0971          .0706         1.5787
   10917.4229862942924.0623           1219.6334     27271.8317

CORRELATION COEFFICIENTS:
 1.000
 -.655  1.000
 -.431   .236  1.000
 -.196   .455   .601  1.000
  .142  -.470  -.282  -.356  1.000
 -.013   .021  -.101  -.128  -.080  1.000

ITERATIONS  FCN2 USED  FCN4 USED   DIVERGED
    5.4420      .3640     0.0000      .6360
```

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0258	.0038	.0025	.0559
.0211	.0028	.0021	.0480
.0389	.1363	.0164	.3672
.0435	.0987	.0139	.3112
-5.1328	27.3535	.0449	1.0040
.0019	.0000	.0002	.0036

CORRELATION COEFFICIENTS:

1.000					
.025	1.000				
-.726	.267	1.000			
-.274	.697	.657	1.000		
-.059	.044	-.039	-.024	1.000	
.052	.103	-.198	-.098	.278	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
6.7940	0.0000	.6480	.0360

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0262	.0255	.0071	.1577
.0172	.0156	.0055	.1238
.1195	.8769	.0415	.9288
.0832	.3962	.0279	.6239
-5.4728	31.3779	.0534	1.1942
.0000	.0000	.0000	.0001

CORRELATION COEFFICIENTS:

1.000					
-.084	1.000				
-.948	.169	1.000			
-.335	.906	.470	1.000		
-.189	.179	.157	.179	1.000	
-.611	.270	.602	.372	.194	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.4840	.0360	.0240	.0020

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0157	.0057	.0033	.0736
.0135	.0044	.0029	.0649
.0673	.3410	.0259	.5800
.0605	.1753	.0185	.4142
-5.4509	31.0091	.0509	1.1387
.0000	.0000	.0000	.0000

CORRELATION COEFFICIENTS:

1.000					
-.156	1.000				
-.925	.248	1.000			
-.465	.836	.621	1.000		
-.174	.172	.123	.155	1.000	
-.484	.301	.616	.538	.083	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.6780	.0060	.0020	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0109	.0031	.0024	.0542
.0093	.0021	.0020	.0444
.0389	.1363	.0164	.3672
.0435	.0987	.0139	.3112
-5.3085	29.5371	.0521	1.1648
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.185	1.000				
-.853	.324	1.000			
-.405	.767	.657	1.000		
-.116	.128	-.021	-.002	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0128	.0024	.0021	.0475
.0120	.0018	.0018	.0402
.0389	.1363	.0164	.3672
.0435	.0987	.0139	.3112
-5.2814	29.1905	.0509	1.1390
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.152	1.000				
-.852	.298	1.000			
-.392	.763	.657	1.000		
-.125	.138	-.008	.004	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.4106	23.8136	.2175	4.8626
-.4928	.3543	.0149	.3337
.0389	.1363	.0164	.3672
.0435	.0987	.0139	.3112
-2.6763	8.0451	.0420	.9394
21832.6118		1569.9964	35106.1868

CORRELATION COEFFICIENTS:

1.000					
-.011	1.000				
-.001	.446	1.000			
.046	.685	.657	1.000		
-.119	-.650	-.637	-.772	1.000	
-.111	.295	.208	.295	-.368	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
1.3280	.0280	0.0000	.9780

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0278	.0042	.0026	.0589
.0241	.0032	.0023	.0514
.0657	.2088	.0202	.4522
.0671	.1855	.0190	.4254
-5.2248	28.3358	.0456	1.0186
.0018	.0000	.0002	.0038

CORRELATION COEFFICIENTS:

1.000					
-.185	1.000				
-.765	.478	1.000			
-.475	.755	.819	1.000		
-.070	.029	-.052	-.030	1.000	
.060	.014	-.191	-.160	.244	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
6.4860	0.0000	.6080	.0360

SAMPLE SIZE: 20 ESTIMATOR: MMF1 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0162	.0136	.0052	.1153
.0194	.0110	.0046	.1032
.1022	.7476	.0384	.8586
.0960	.4340	.0291	.6518
-5.5459	32.0869	.0516	1.1530
.0001	.0000	.0000	.0004

CORRELATION COEFFICIENTS:

1.000					
-.197	1.000				
-.953	.317	1.000			
-.462	.893	.608	1.000		
-.154	.207	.122	.169	1.000	
-.501	.220	.367	.245	.114	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.1040	.0260	.0220	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0146	.0081	.0040	.0886
.0153	.0034	.0025	.0562
.0807	.4922	.0312	.6969
.0714	.2126	.0204	.4556
-5.5241	31.7455	.0496	1.1091
.0000	.0000	.0000	.0000

CORRELATION COEFFICIENTS:

1.000					
-.264	1.000				
-.946	.392	1.000			
-.509	.857	.677	1.000		
-.131	.196	.095	.138	1.000	
-.645	.202	.628	.367	.065	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.7380	.0060	.0040	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0110	.0034	.0026	.0572
.0128	.0028	.0023	.0517
.0657	.2088	.0202	.4522
.0671	.1855	.0190	.4254
-5.3826	30.3372	.0522	1.1681
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.417	1.000				
-.868	.601	1.000			
-.582	.858	.819	1.000		
-.091	.105	-.018	-.003	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMF4 SEED #: [illegible]
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0132	.0027	.0022	.0498
.0149	.0023	.0020	.0454
.0657	.2088	.0202	.4522
.0671	.1855	.0190	.4254
-5.3476	29.8586	.0502	1.1233
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.376	1.000				
-.861	.579	1.000			
-.560	.851	.819	1.000		
-.107	.119	-.003	.011	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: .500 TRUE Q: .500

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1855	6.0348	.1095	2.4496
-.5038	.3749	.0156	.3480
.0657	.2088	.0202	.4522
.0671	.1855	.0190	.4254
-2.6947	8.2702	.0449	1.0044
20088.0592		1550.7309	34675.3961

CORRELATION COEFFICIENTS:

1.000					
-.087	1.000				
.010	.475	1.000			
.022	.619	.819	1.000		
-.099	-.691	-.579	-.658	1.000	
-.151	.335	.154	.239	-.407	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
1.6860	.0460	0.0000	.9600

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0206	.0020	.0018	.0400
.0075	.0351	.0084	.1872
.0596	.1885	.0192	.4301
.1283	.9428	.0430	.9625
-5.1526	27.7626	.0493	1.1017
.0027	.0003	.0007	.0157

CORRELATION COEFFICIENTS:

1.000					
-.102	1.000				
-.823	.281	1.000			
-.366	.739	.653	1.000		
-.126	-.093	-.015	-.044	1.000	
.026	.631	.006	.362	.022	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.1300	0.0000	.5500	.0460

SAMPLE SIZE: 20 ESTIMATOR: MMF1 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0096	.0060	.0034	.0769
.1646	.1649	.0166	.3712
.1109	.5376	.0324	.7247
.6537	3.8276	.0825	1.8440
-5.4101	30.6253	.0521	1.1644
.0010	.0000	.0002	.0041

CORRELATION COEFFICIENTS:

1.000					
-.095	1.000				
-.904	.329	1.000			
-.236	.906	.517	1.000		
-.173	.028	.142	.090	1.000	
-.038	.193	.052	.146	.180	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
21.4620	.3040	.2900	.0360

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0109	.0040	.0028	.0623
.1951	.3358	.0244	.5457
.1402	.5198	.0316	.7072
.9048	7.0645	.1118	2.4992
-5.4613	31.1536	.0515	1.1522
.0001	.0000	.0000	.0004

CORRELATION COEFFICIENTS:

1.000					
-.051	1.000				
-.929	.212	1.000			
-.202	.911	.424	1.000		
-.197	.188	.196	.225	1.000	
-.400	.048	.335	.075	.075	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
9.1860	.0920	.0580	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0080	.0014	.0017	.0372
.0432	.0516	.0100	.2229
.0596	.1885	.0192	.4301
.1283	.9428	.0430	.9625
-5.2792	29.3794	.0549	1.2285
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.166	1.000				
-.880	.344	1.000			
-.411	.832	.653	1.000		
-.207	-.069	.095	-.051	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0083	.0011	.0014	.0323
.0321	.0372	.0085	.1903
.0596	.1885	.0192	.4301
.1283	.9428	.0430	.9625
-5.2419	28.9400	.0541	1.2095
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.145	1.000				
-.882	.323	1.000			
-.402	.799	.653	1.000		
-.206	-.052	.092	-.027	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1466	17.3369	.1861	4.1612
-.2254	.1996	.0172	.3857
.0596	.1885	.0192	.4301
.1283	.9428	.0430	.9625
-3.7172	15.6440	.0604	1.3515
31422.5327		1820.9039	40716.6486

CORRELATION COEFFICIENTS:

1.000					
-.022	1.000				
.017	.434	1.000			
.021	.728	.653	1.000		
-.078	-.717	-.363	-.477	1.000	
-.078	.347	.030	.140	-.453	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.1360	.0900	0.0000	.9140

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0192	.0017	.0016	.0360
.0154	.0990	.0141	.3143
.0612	.4032	.0283	.6320
.3085	32.9894	.2565	5.7354
-5.1930	28.2892	.0514	1.1499
.0025	.0000	.0003	.0058

CORRELATION COEFFICIENTS:

1.000					
-.276	1.000				
-.596	.832	1.000			
-.305	.923	.886	1.000		
-.104	.020	.010	.063	1.000	
.176	.036	-.158	-.033	.203	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.3340	0.0000	.5680	.0760

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0052	.0007	.0012	.0260
.1764	.1860	.0176	.3935
.0941	.2023	.0197	.4398
.6735	3.7441	.0811	1.8140
-5.4562	31.1534	.0526	1.1761
.0007	.0000	.0001	.0024

CORRELATION COEFFICIENTS:

1.000					
-.338	1.000				
-.841	.551	1.000			
-.516	.924	.753	1.000		
-.152	-.039	.073	.025	1.000	
.050	.159	-.041	.090	.210	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
20.3400	.3000	.2520	.0240

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0056	.0005	.0009	.0212
.1703	.3117	.0238	.5317
.0863	.1568	.0173	.3865
.7146	5.6113	.1010	2.2585
-5.5259	31.9064	.0524	1.1706
.0000	.0000	.0000	.0000

CORRELATION COEFFICIENTS:

1.000					
-.162	1.000				
-.861	.340	1.000			
-.299	.957	.506	1.000		
-.168	.114	.115	.131	1.000	
-.331	-.087	.376	-.040	-.057	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
7.9620	.0780	.0480	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0063	.0009	.0013	.0287
.0498	.1680	.0182	.4069
.0612	.4032	.0283	.6320
.3085	32.9894	.2565	5.7354
-5.2775	29.3751	.0552	1.2340
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.627	1.000				
-.870	.837	1.000			
-.648	.923	.886	1.000		
-.213	.014	.085	.057	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0067	.0006	.0011	.0242
.0386	.1253	.0157	.3519
.0612	.4032	.0283	.6320
.3085	32.9894	.2565	5.7354
-5.2489	29.0237	.0543	1.2136
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.557	1.000				
-.832	.831	1.000			
-.579	.921	.886	1.000		
-.201	.029	.082	.064	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: .500 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0932	3.0287	.0777	1.7378
-.2162	.2443	.0199	.4445
.0612	.4032	.0283	.6320
.3085	32.9894	.2565	5.7354
-3.6829	15.4877	.0620	1.3870
34277.0325		1890.6544	42276.3165

CORRELATION COEFFICIENTS:

1.000					
-.002	1.000				
.017	.755	1.000			
.002	.707	.886	1.000		
-.110	-.593	-.225	-.056	1.000	
-.086	.258	.018	-.015	-.402	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
2.0820	.0840	0.0000	.9260

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0073	.0006	.0011	.0235
-.1244	.0977	.0128	.2867
-.0445	.1594	.0177	.3968
-.3738	2.9449	.0749	1.6749
-4.8838	25.0423	.0488	1.0914
.0033	.0001	.0003	.0074

CORRELATION COEFFICIENTS:

1.000					
-.187	1.000				
-.841	.339	1.000			
-.432	.731	.704	1.000		
-.087	-.133	-.111	-.106	1.000	
.176	.136	-.216	-.031	.320	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.9080	0.0000	.6240	.1000

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0001	.0000	.0003	.0061
.0445	.2011	.0200	.4462
-.0753	.0746	.0117	.2625
-.0387	3.6830	.0858	1.9187
-5.0910	27.5542	.0572	1.2791
.0030	.0000	.0002	.0053

CORRELATION COEFFICIENTS:

1.000					
-.230	1.000				
-.658	.465	1.000			
-.453	.842	.754	1.000		
-.024	-.217	-.273	-.237	1.000	
.111	.093	-.287	-.124	.334	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
42.8940	.7100	.6620	.1340

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0033	.0002	.0006	.0142
.5931	1.3963	.0457	1.0220
.0845	.1502	.0169	.3782
2.6105	22.6746	.1781	3.9824
-5.4676	31.1032	.0492	1.0993
.0001	.0000	.0000	.0005

CORRELATION COEFFICIENTS:

1.000					
-.033	1.000				
-.868	.212	1.000			
-.216	.901	.470	1.000		
-.194	-.174	.111	-.082	1.000	
.026	.088	-.070	.019	.123	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
23.0400	.4200	.3120	.0020

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0027	.0004	.0009	.0205
-.0156	.1677	.0183	.4092
-.0445	.1594	.0177	.3968
-.3738	2.9449	.0749	1.6749
-4.9935	26.4094	.0543	1.2143
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.229	1.000				
-.881	.323	1.000			
-.549	.753	.704	1.000		
-.214	-.069	.054	-.040	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMF4 SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0027	.0003	.0008	.0174
-.0552	.1256	.0157	.3501
-.0445	.1594	.0177	.3968
-.3738	2.9449	.0749	1.6749
-5.0137	26.6831	.0556	1.2433
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.224	1.000				
-.887	.317	1.000			
-.542	.738	.704	1.000		
-.193	-.084	.018	-.063	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0297	.0765	.0123	.2750
-.1896	.2043	.0183	.4102
-.0445	.1594	.0177	.3968
-.3738	2.9449	.0749	1.6749
-3.7504	15.8421	.0596	1.3330
43889.7862		1967.1945	43987.8052

CORRELATION COEFFICIENTS:

1.000					
-.038	1.000				
-.073	.367	1.000			
-.048	.718	.704	1.000		
.086	-.650	-.253	-.393	1.000	
.078	.395	.017	.147	-.500	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
1.9980	.0560	0.0000	.9520

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0078	.0004	.0008	.0182
-.1214	.1016	.0132	.2948
-.0481	.0974	.0138	.3084
-.4082	3.1141	.0768	1.7168
-4.9129	25.5037	.0523	1.1692
.0044	.0001	.0004	.0098

CORRELATION COEFFICIENTS:

1.000					
-.101	1.000				
-.732	.294	1.000			
-.368	.714	.706	1.000		
.004	-.053	-.248	-.151	1.000	
.129	.239	-.199	.067	.264	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.1260	0.0000	.5960	.1360

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0007	.0001	.0005	.0108
.0461	.1732	.0185	.4137
-.0493	.1070	.0145	.3234
-.0183	3.5246	.0840	1.8773
-5.1588	28.3293	.0586	1.3099
.0030	.0000	.0003	.0057

CORRELATION COEFFICIENTS:

1.000					
-.212	1.000				
-.786	.404	1.000			
-.464	.811	.734	1.000		
-.112	-.109	-.136	-.153	1.000	
.076	.052	-.259	-.160	.292	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
42.2360	.6980	.6360	.1380

AD-A138 062 PARAMETER ESTIMATION FOR THE FOUR PARAMETER BETA DISTRIBUTION(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING B R TREAT DEC 83 2/2

UNCLASSIFIED AFIT/GOR/ENC/83D-6 F/G 12/1 NL

END

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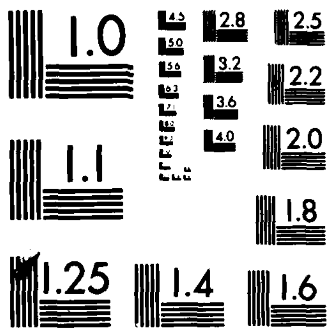

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0023	.0001	.0003	.0073
.4903	1.1993	.0438	.9793
.0671	.0844	.0126	.2827
2.0920	18.3032	.1669	3.7319
-5.4615	31.0662	.0498	1.1126
.0001	.0000	.0000	.0009

CORRELATION COEFFICIENTS:

1.000					
-.072	1.000				
-.766	.255	1.000			
-.276	.890	.545	1.000		
-.155	-.060	.035	-.058	1.000	
.027	.265	-.060	.119	.056	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
20.8700	.3400	.2520	.0020

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0015	.0001	.0004	.0099
-.0155	.1749	.0187	.4179
-.0481	.0974	.0138	.3084
-.4082	3.1141	.0768	1.7168
-4.9077	25.5365	.0539	1.2046
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.241	1.000				
-.814	.289	1.000			
-.577	.725	.706	1.000		
-.209	-.001	-.011	-.037	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0017	.0001	.0004	.0088
-.0538	.1299	.0159	.3564
-.0481	.0974	.0138	.3084
-.4082	3.1141	.0768	1.7168
-4.9433	25.9418	.0549	1.2270
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.204	1.000				
-.820	.275	1.000			
-.536	.711	.706	1.000		
-.168	-.026	-.065	-.076	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: .500 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0183	.3045	.0247	.5515
-.2020	.2301	.0195	.4351
-.0481	.0974	.0138	.3084
-.4082	3.1141	.0768	1.7168
-3.6242	14.8866	.0592	1.3236
41287.0150		1918.4298	42897.3948

CORRELATION COEFFICIENTS:

1.000					
-.238	1.000				
-.004	.370	1.000			
-.003	.657	.706	1.000		
-.001	-.627	-.397	-.418	1.000	
-.010	.347	.144	.125	-.503	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
1.7660	.0480	0.0000	.9540

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0347	.0477	.0096	.2156
.0154	.0105	.0045	.1014
.1568	1.1819	.0481	1.0758
.1269	1.0490	.0455	1.0163
-5.4349	31.0255	.0545	1.2197
.0035	.0014	.0016	.0367

CORRELATION COEFFICIENTS:

1.000					
-.056	1.000				
-.715	.396	1.000			
-.177	.777	.679	1.000		
-.116	-.029	.067	.012	1.000	
-.813	.003	.344	-.032	.107	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.2300	0.0000	.4780	.0500

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1219	.1168	.0143	.3193
.1329	.1240	.0146	.3261
.9154	5.9807	.1014	2.2678
.9984	6.9690	.1093	2.4438
-5.4968	31.5701	.0521	1.1641
.0004	.0000	.0001	.0021

CORRELATION COEFFICIENTS:

1.000					
-.045	1.000				
-.915	.253	1.000			
-.218	.918	.482	1.000		
-.196	.209	.177	.186	1.000	
-.184	.116	.143	.106	.202	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
18.4160	.2400	.2100	.0140

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0682	.0660	.0111	.2476
.0530	.0399	.0086	.1927
.5494	3.8385	.0841	1.8806
.4689	2.9264	.0736	1.6452
-5.5735	32.4801	.0532	1.1901
.0002	.0000	.0001	.0019

CORRELATION COEFFICIENTS:

1.000					
-.130	1.000				
-.904	.300	1.000			
-.323	.874	.570	1.000		
-.208	.157	.201	.173	1.000	
-.366	.023	.309	.054	.094	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
7.6480	.0660	.0400	.0020

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0369	.0428	.0091	.2035
.0265	.0160	.0055	.1238
.1568	1.1819	.0481	1.0758
.1269	1.0490	.0455	1.0163
-5.5422	32.2943	.0562	1.2564
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.176	1.000				
-.838	.470	1.000			
-.297	.864	.679	1.000		
-.054	-.080	-.041	-.120	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0298	.0306	.0077	.1724
.0210	.0120	.0048	.1075
.1568	1.1819	.0481	1.0758
.1269	1.0490	.0455	1.0163
-5.4518	31.2843	.0559	1.2499
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.154	1.000				
-.830	.446	1.000			
-.289	.838	.679	1.000		
-.095	-.030	.026	-.055	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0713	1.1953	.0488	1.0910
-.1913	.1006	.0113	.2529
.1568	1.1819	.0481	1.0758
.1269	1.0490	.0455	1.0163
-4.3622	21.3271	.0678	1.5161
28004.4527		1738.0204	38863.3171

CORRELATION COEFFICIENTS:

1.000					
.091	1.000				
-.075	.469	1.000			
.013	.706	.679	1.000		
-.135	-.763	-.254	-.305	1.000	
-.002	.028	-.139	-.166	-.128	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.9640	.2920	0.0000	.7200

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0182	.0126	.0049	.1106
.0220	.0152	.0054	.1214
.1359	1.0922	.0463	1.0362
.1489	1.2005	.0485	1.0855
-5.4383	30.9867	.0531	1.1880
.0045	.0016	.0018	.0400

CORRELATION COEFFICIENTS:

1.000					
-.299	1.000				
-.797	.571	1.000			
-.508	.873	.798	1.000		
.023	-.027	-.030	-.031	1.000	
-.338	.335	.474	.448	.043	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.4680	0.0000	.4780	.0660

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1308	.1275	.0149	.3322
.1353	.1300	.0149	.3343
1.0458	6.9633	.1083	2.4227
1.0345	7.1689	.1104	2.4695
-5.5429	32.1327	.0531	1.1870
.0003	.0000	.0001	.0014

CORRELATION COEFFICIENTS:

1.000					
-.108	1.000				
-.921	.299	1.000			
-.292	.927	.533	1.000		
-.161	.123	.142	.097	1.000	
-.219	.284	.282	.317	.047	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
18.7440	.2540	.2320	.0060

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0603	.0537	.0100	.2237
.0515	.0347	.0080	.1790
.5558	3.9085	.0848	1.8973
.4669	3.0423	.0752	1.6806
-5.6194	33.0594	.0544	1.2170
.0001	.0000	.0000	.0004

CORRELATION COEFFICIENTS:

1.000					
-.241	1.000				
-.912	.416	1.000			
-.429	.910	.650	1.000		
-.134	.084	.147	.103	1.000	
-.291	.167	.387	.280	.036	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
7.9640	.0700	.0400	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0223	.0166	.0057	.1268
.0315	.0210	.0063	.1415
.1359	1.0922	.0463	1.0362
.1489	1.2005	.0485	1.0855
-5.5397	32.2009	.0550	1.2298
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.385	1.000				
-.841	.555	1.000			
-.556	.884	.798	1.000		
.162	-.148	-.165	-.174	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0167	.0125	.0049	.1106
.0256	.0157	.0055	.1229
.1359	1.0922	.0463	1.0362
.1489	1.2005	.0485	1.0855
-5.4371	31.0041	.0537	1.2008
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.350	1.000				
-.811	.535	1.000			
-.529	.864	.798	1.000		
.095	-.079	-.085	-.093	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: 1.000 TRUE Q: 1.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0068	.3662	.0271	.6051
-.2039	.1318	.0134	.3004
.1359	1.0922	.0463	1.0362
.1489	1.2005	.0485	1.0855
-4.2849	20.5386	.0660	1.4760
24256.0830		1644.8470	36779.8976

CORRELATION COEFFICIENTS:

1.000					
-.144	1.000				
-.089	.521	1.000			
-.043	.666	.798	1.000		
-.073	-.710	-.330	-.341	1.000	
-.064	.017	-.145	-.187	-.158	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.7960	.3140	0.0000	.7160

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0109	.0042	.0029	.0641
.0404	1.3558	.0520	1.1637
.0390	1.0688	.0462	1.0331
.6198	85.3886	.4123	9.2198
-5.3458	30.0865	.0549	1.2283
.0052	.0007	.0012	.0268

CORRELATION COEFFICIENTS:

1.000					
-.117	1.000				
-.806	.218	1.000			
-.212	.976	.361	1.000		
.050	-.030	-.089	-.030	1.000	
.093	.038	-.106	-.001	.224	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.5700	0.0000	.4960	.1400

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MFAN	MEAN SQUARF	STD ERROR	STD DEV
-.0169	.0076	.0038	.0854
.3020	.3350	.0221	.4937
.2379	1.1448	.0467	1.0432
1.5669	10.4551	.1265	2.8284
-5.5454	32.1465	.0528	1.1813
.0018	.0000	.0002	.0055

CORRELATION COEFFICIENTS:

1.000					
-.062	1.000				
-.860	.279	1.000			
-.274	.866	.585	1.000		
-.094	-.132	.037	-.096	1.000	
.056	.126	-.083	.015	.135	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
35.0200	.5720	.5020	.0800

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0266	.0098	.0043	.0954
.1852	.2990	.0230	.5144
.3757	2.0031	.0610	1.3645
1.4175	12.5278	.1450	3.2432
-5.5884	32.5717	.0518	1.1583
.0001	.0000	.0000	.0001

CORRELATION COEFFICIENTS:

1.000					
-.119	1.000				
-.926	.228	1.000			
-.376	.882	.542	1.000		
-.145	-.028	.137	.020	1.000	
-.068	.002	.074	.029	-.016	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
16.6900	.2500	.1680	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0130	.0053	.0032	.0719
.1323	2.5039	.0705	1.5768
.0390	1.0688	.0462	1.0331
.6198	85.3886	.4123	9.2198
-5.3159	29.5850	.0515	1.1517
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.156	1.000				
-.890	.214	1.000			
-.265	.975	.361	1.000		
-.083	-.069	-.001	-.068	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0099	.0039	.0028	.0617
.0921	2.0057	.0632	1.4132
.0390	1.0688	.0462	1.0331
.6198	85.3886	.4123	9.2198
-5.3062	29.5796	.0534	1.1934
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.159	1.000				
-.868	.210	1.000			
-.268	.976	.361	1.000		
-.059	-.087	-.010	-.085	1.000	
					1 [illegible]

ITERATIONS	FCN2 USED	FCN4 USED	DI[illegible]GED
0.0000	0.0000	0.0000	[illegible]000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0214	.0442	.0093	.2091
-.0285	1.3855	.0526	1.1767
.0390	1.0688	.0462	1.0331
.6198	85.3886	.4123	9.2198
-5.1479	28.5082	.0634	1.4169
30686.7514		1753.7476	39214.9883

CORRELATION COEFFICIENTS:

1.000					
-.064	1.000				
-.257	.231	1.000			
-.078	.967	.361	1.000		
.009	-.109	-.076	-.043	1.000	
.052	-.050	-.290	-.104	-.103	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
4.6420	.2980	0.0000	.7200

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0226	.0225	.0066	.1482
-.0211	.2312	.0215	.4804
.0983	1.6641	.0575	1.2863
.2115	12.7315	.1593	3.5618
-5.3488	30.0478	.0536	1.1991
.0298	.1043	.0144	.3216

CORRELATION COEFFICIENTS:

1.000					
-.195	1.000				
-.761	.312	1.000			
-.273	.635	.684	1.000		
-.035	.013	-.035	-.032	1.000	
-.693	.542	.322	.089	.098	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.0140	0.0000	.4760	.1260

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0198	.0091	.0042	.0935
.2941	.3286	.0220	.4921
.2705	1.3714	.0510	1.1394
1.6545	11.2141	.1302	2.9115
-5.5923	32.7880	.0550	1.2306
.0017	.0000	.0002	.0052

CORRELATION COEFFICIENTS:

1.000					
-.073	1.000				
-.903	.278	1.000			
-.271	.882	.538	1.000		
-.078	-.202	.038	-.138	1.000	
.050	.228	-.050	.086	.019	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
34.9520	.5720	.5240	.0500

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0361	.0076	.0037	.0830
.1735	.2675	.0218	.4873
.3748	1.6846	.0556	1.2426
1.5168	13.2541	.1480	3.3096
-5.6374	33.3066	.0553	1.2356
.0001	.0000	.0000	.0001

CORRELATION COEFFICIENTS:

1.000					
-.169	1.000				
-.914	.311	1.000			
-.410	.888	.602	1.000		
-.065	-.083	.082	-.001	1.000	
-.238	-.140	.199	-.110	-.013	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
17.2780	.2760	.1680	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0208	.0154	.0055	.1222
.0369	.1981	.0198	.4436
.0983	1.6641	.0575	1.2863
.2115	12.7315	.1593	3.5618
-5.3743	30.3758	.0546	1.2216
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.253	1.000				
-.874	.446	1.000			
-.394	.832	.684	1.000		
-.129	-.118	.043	-.092	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0168	.0114	.0047	.1054
.0065	.1487	.0172	.3855
.0983	1.6641	.0575	1.2863
.2115	12.7315	.1593	3.5618
-5.3485	30.1883	.0562	1.2577
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.241	1.000				
-.863	.445	1.000			
-.379	.829	.684	1.000		
-.130	-.088	.054	-.058	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: 1.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0764	4.4707	.0945	2.1130
-.1121	.1907	.0189	.4221
.0983	1.6641	.0575	1.2863
.2115	12.7315	.1593	3.5618
-5.1000	28.0906	.0645	1.4424
28302.8705		1716.7708	38388.1612

CORRELATION COEFFICIENTS:

1.000					
-.010	1.000				
-.028	.416	1.000			
-.001	.719	.684	1.000		
-.118	-.358	-.066	-.092	1.000	
-.078	-.003	-.238	-.151	-.052	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
5.3320	.3300	0.0000	.6980

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0216	.1062	.0145	.3252
.0558	.4299	.0292	.6533
.5097	28.7623	.2388	5.3388
1.2532	227.7757	.6726	15.0401
-5.5640	32.3935	.0536	1.1979
.0045	.0010	.0014	.0312

CORRELATION COEFFICIENTS:

1.000					
-.121	1.000				
-.936	.247	1.000			
-.125	.975	.248	1.000		
.022	.015	-.020	.024	1.000	
-.175	.089	.129	.041	-.108	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.8620	0.0000	.2780	.0540

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.2139	.2098	.0181	.4050
.2466	.2209	.0179	.4001
2.0271	14.9516	.1473	3.2928
2.1991	14.9218	.1420	3.1758
-5.5681	32.2754	.0504	1.1277
.0010	.0000	.0001	.0019

CORRELATION COEFFICIENTS:

1.000					
.249	1.000				
-.910	-.058	1.000			
.033	.889	.248	1.000		
.055	.001	-.145	-.098	1.000	
-.119	.251	.109	.185	.155	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
40.2420	.6300	.5920	.0160

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1360	.1276	.0148	.3302
.0890	.0782	.0119	.2652
1.4540	11.8254	.1394	3.1163
1.1962	9.3558	.1259	2.8151
-5.6575	33.2908	.0507	1.1332
.0002	.0000	.0000	.0007

CORRELATION COEFFICIENTS:

1.000					
-.058	1.000				
-.914	.262	1.000			
-.270	.895	.534	1.000		
.021	.030	-.045	.024	1.000	
-.413	.052	.409	.143	-.020	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
21.0820	.2720	.2040	.0020

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0611	.1820	.0189	.4222
.1106	.6878	.0368	.8220
.5097	28.7623	.2388	5.3388
1.2532	227.7757	.6726	15.0401
-5.6148	32.8994	.0524	1.1721
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.136	1.000				
-.949	.250	1.000			
-.135	.972	.248	1.000		
.166	-.046	-.171	-.027	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0392	.1412	.0167	.3736
.0870	.6840	.0368	.8224
.5097	28.7623	.2388	5.3388
1.2532	227.7757	.6726	15.0401
-5.5339	32.0356	.0531	1.1881
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.134	1.000				
-.949	.237	1.000			
-.147	.983	.248	1.000		
.111	-.009	-.114	-.002	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 1
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0112	.1029	.0143	.3206
-.0193	.4353	.0295	.6595
.5097	28.7623	.2388	5.3388
1.2532	227.7757	.6726	15.0401
-5.2134	29.0881	.0618	1.3813
15048.0749		1408.5730	31496.6499

CORRELATION COEFFICIENTS:

1.000					
-.136	1.000				
-.925	.258	1.000			
-.126	.957	.248	1.000		
.069	-.127	-.055	-.003	1.000	
.116	-.146	-.133	-.075	.091	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
8.1460	.6600	0.0000	.3640

SAMPLE SIZE: 20 ESTIMATOR: FULLM SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0038	.0899	.0134	.2999
.3141	43.0766	.2932	6.5558
.6922	56.7612	.3355	7.5021
18.2458	152169.8836	17.4262	389.6626
-5.4999	31.5087	.0502	1.1223
.0061	.0069	.0037	.0831

CORRELATION COEFFICIENTS:

1.000					
-.272	1.000				
-.735	.795	1.000			
-.265	.999	.786	1.000		
.085	-.026	-.056	-.024	1.000	
-.233	-.003	.043	-.003	.025	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
3.9240	0.0000	.3220	.0620

SAMPLE SIZE: 20 ESTIMATOR: MME1 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1900	.1890	.0175	.3910
.2410	.2164	.0178	.3979
1.9542	14.0902	.1433	3.2049
2.2522	15.9038	.1472	3.2911
-5.5443	31.9206	.0486	1.0867
.0010	.0000	.0001	.0016

CORRELATION COEFFICIENTS:

1.000					
.222	1.000				
-.893	-.021	1.000			
.034	.889	.269	1.000		
.014	-.064	-.077	-.108	1.000	
-.102	.364	.099	.260	.124	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
40.1460	.6320	.5800	.0120

SAMPLE SIZE: 20 ESTIMATOR: MME2 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.1149	.1148	.0143	.3187
.0844	.0777	.0119	.2656
1.4054	11.6395	.1390	3.1088
1.2249	9.8488	.1292	2.8894
-5.6456	33.1488	.0505	1.1294
.0002	.0000	.0000	.0005

CORRELATION COEFFICIENTS:

1.000					
-.107	1.000				
-.892	.315	1.000			
-.291	.886	.577	1.000		
-.015	.010	.003	.020	1.000	
-.457	.025	.441	.134	-.016	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
21.9180	.2880	.2060	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME3 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0436	.1428	.0168	.3754
.4385	68.2349	.3689	8.2488
.6922	56.7612	.3355	7.5021
18.2458	152169.8836	17.4262	389.6626
-5.6116	32.9469	.0540	1.2072
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.283	1.000				
-.759	.796	1.000			
-.274	.999	.786	1.000		
.173	-.045	-.156	-.039	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MME4 SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
-.0223	.1079	.0147	.3278
.3017	32.5249	.2547	5.6951
.6922	56.7612	.3355	7.5021
18.2458	152169.8836	17.4262	389.6626
-5.5075	31.8319	.0548	1.2246
0.0000	0.0000	0.0000	0.0000

CORRELATION COEFFICIENTS:

1.000					
-.252	1.000				
-.735	.801	1.000			
-.238	.999	.786	1.000		
.090	.117	.008	.122	1.000	
					1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
0.0000	0.0000	0.0000	0.0000

SAMPLE SIZE: 20 ESTIMATOR: MMLE SEED #: 2
TRUE P: 2.000 TRUE Q: 2.000

MEAN	MEAN SQUARE	STD ERROR	STD DEV
.0252	.0941	.0137	.3058
.1978	33.4164	.2584	5.7773
.6922	56.7612	.3355	7.5021
18.2458	152169.8836	17.4262	389.6626
-5.1391	28.6337	.0667	1.4912
16148.3959		1476.9256	33025.0601

CORRELATION COEFFICIENTS:

1.000					
-.238	1.000				
-.686	.799	1.000			
-.225	.999	.786	1.000		
.143	-.026	-.071	-.007	1.000	
.087	-.035	-.109	-.024	.036	1.000

ITERATIONS	FCN2 USED	FCN4 USED	DIVERGED
7.7260	.6240	0.0000	.4060

APPENDIX B

FORTRAN SOURCE CODE

```
      PROGRAM BETA
      PARAMETER (MAXS=20)
      PARAMETER (NMAX=500)
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      REAL ARRAY(NMAX,6)
      REAL STATS(8)
      REAL WK(6)
      REAL SAMPLE(MAXS)
      COMMON /DATUM/SAMPLE
      INTEGER NBR(5)
      DOUBLE PRECISION SEED(2),DSEED
      DATA SEED/4813176891101.D0,
     +        228262754377251.D0/
100   READ(5,110,END=7000,ERR=8000)NREPS,N,PTRUE,QTRUE,
     +    IEST,NSEED
      WRITE(6,105)
105   FORMAT(1X)
      WRITE(6,110)NREPS,N,PTRUE,QTRUE,IEST,NSEED
110   FORMAT(2I4,2F5.2,2I3)


C     CHECK FOR INPUT ERROR
      IF (NREPS.LT.1 .OR. N.LT.1 .OR.
     +    IEST.GT.6 .OR. IEST.LT.1 .OR.
     +    PTRUE.LE.0.0 .OR. QTRUE.LE.0.0
     + .OR. NREPS.GT.NMAX
     + .OR. N.GT.MAXS)GO TO 8000
      CALL HELPER(0,DUM,DUM,DUM,DUM)
      CALL MOMNTS(M1,M2,A3,A4,
     +     0.0,1.0,PTRUE,QTRUE)
      WRITE(7,210)N,NREPS,IEST,NSEED
210   FORMAT('1'//' SAMPLE SIZE:',I5,
     +   '   REPETITIONS:',I5/
     +   '   ESTIMATE #:',I5,
     +   '   SEED #:',I5)
      CALL COUNT(0,1)
      DSEED = SEED(NSEED)


      DO 200 I = 1,NREPS
C     GENERATE A SORTED SAMPLE
      CALL GGBTR(DSEED,PTRUE,QTRUE,N,SAMPLE)
      CALL VSRTA(SAMPLE,N)


C     CALCULATE MOMENTS
```

```
C     SET UP FOR CALL TO BESTAT
      NBR(1) = 1
      NBR(2) = N
      NBR(3) = N
      NBR(4) = 1
      NBR(5) = 0
      WT=-1.0
      CALL BESTAT(SAMPLE,N,WT,NBR,DUM,
     +     STATS,8,WK,IER)
      IF (IER.NE.0) GO TO 8000
      M1 = STATS(1)
      M2 = STATS(2)
      A3 = STATS(4)
      A4 = 3.0 + STATS(5)
      SDEV = SQRT(M2)
      BOT = SAMPLE(1)
      TOP = SAMPLE(N)
      SPAN = TOP - BOT
      CALL MODEST(1,ARRAY,NMAX)
      IF (IEST.NE.1)
     +    CALL MODEST(IEST,ARRAY,NMAX)
200   CONTINUE



C     OUTPUT RESULTS
      RNR = FLOAT(NREPS)
      CALL FINIS(IEST,ARRAY,NMAX)
C     GO BACK AND DO THAT AGAIN.
      GO TO 100
C     INPUT RAN OUT --- STOP NOW
7000  CONTINUE
      GO TO 9000



C     ERROR. ERROR.
8000  CONTINUE
      WRITE(6,8010)IER
8010  FORMAT(' -BETA- IER=',I3)
9000  CONTINUE
      END
```

```
      SUBROUTINE ABPQ(IEST,A,B,P,Q)
      INTEGER IEST
      REAL A,B,P,Q
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      GO TO (100,200,300,200,300),IEST
100   CONTINUE
      WRITE(6,77)IEST
77    FORMAT(' -ABPQ- IEST =',I7)
      RETURN
C     IEST=2,4 GIVES A,B CALCULATED FROM P,Q
C     USING E(X1:A,B,P,Q) = 1/(N+1) AND
C           E(XN:A,B,P,Q) = N/(N+1).
200   CONTINUE
      RN = FLOAT(N)
      CALL MYBETI(RN/(RN+1.0),P,Q,YN,IER)
      CALL MYBETI(1.0/(RN+1.0),P,Q,Y1,IER)
      RNGE = SPAN/(YN-Y1)
      A = BOT - RNGE*Y1
      B = A + RNGE
      RETURN
C     IEST=3,5 GIVES A,B FROM P,Q USING
C     E(X1:A,B,P,Q) = X(1)
C           AND
C     E(XN:A,B,P,Q) = X(N)
300   CONTINUE
      XN=XNMEAN(N,P,Q)
      X1=1.0-XNMEAN(N,Q,P)
      ESPAN = XN - X1
      A=BOT-X1*SPAN/ESPAN
      B=TOP+(1.0-XN)*SPAN/ESPAN
      RETURN
      END
```

```
      SUBROUTINE A3A4(F1,F2,P,Q)
      REAL F1,F2,P,Q,DUM1,DUM2,A3P,A4P
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      CALL MOMNTS(DUM1,DUM2,A3P,A4P,
     +      0.1,1.0,P,Q)
      F1 = A3-A3P
      F2 = A4-A4P
      RETURN
      END
```

```
      SUBROUTINE BETTER
      REAL A,B,P,Q,FNOW
      COMMON /ESTS/A,B,P,Q,FNOW
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      INTRVL = INTRVL + 1
      IF (FNOW.GE.FN1) RETURN
      A1 = A
      B1 = B
      P1 = P
      Q1 = Q
      FN1 = FNOW
      INTRVL = 0
      RETURN
      END
```

```
      SUBROUTINE CHECKR(J,A,B,P,Q,F,IER)
      REAL A,B,P,Q
      IF (IER.NE.0 .OR. F.GT.1.E-2)
     +     CALL COUNT(J,4)
      RETURN
      END
```

```
      SUBROUTINE COUNT(II,J)
      PARAMETER (NUMEST=6)
      PARAMETER (NUMCNT=4)
      REAL CNTR(NUMEST,NUMCNT)
      SAVE CNTR
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      IF (II.GT.NUMEST .OR. J.GT.NUMCNT)GO TO 9000
      IF (II.GT.0) GO TO 100
      DO 50 K=1,NUMEST
      DO 50 L=1,NUMCNT
50    CNTR(K,L) = 0.0
      RETURN
100   IF (J.GT.0)GO TO 200
      RN = FLOAT(NREPS)
      WRITE(7,150)(CNTR(II,L)/RN,L=1,NUMCNT)
150   FORMAT(' ITERATIONS  FCN2 USED  FCN4 USED   DIVERGED'
     +   /10F11.4)
      RETURN
200   CNTR(II,J) = CNTR(II,J) + 1.0
      RETURN
9000  WRITE(6,9010)II,J
9010  FORMAT(' -COUNT- II =',I7,'  J=',I7)
      RETURN
      END
```

```
      SUBROUTINE CVMDST(DISTNS)
      REAL A1,B1,P1,Q1,FN1
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      S=0.0
      ESPAN = B1 - A1
      DO 1000 J=1,12
      CALL HELPER(J,U,V,X1,X2)
      CALL MYBETA((X1-A1)/ESPAN,P1,Q1,F1,IER)
      CALL MYBETA((X2-A1)/ESPAN,P1,Q1,F2,IER)
      F1 = (F1-U)*(F1-U)
      F2 = (F2-V)*(F2-V)
      S = S + WT24(J)*(F1 + F2)
 1000 CONTINUE
      DISTNS = TLOG(S/2.0)
      RETURN
      END
```

```
      SUBROUTINE EXNEX1(F1,F2,A,B,P,Q,N)
      REAL F1,F2,A,B,P,Q,X1,XN
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      XN = XNMEAN(N,P,Q)
      X1 = 1.0 - XNMEAN(N,Q,P)
      ESPAN = XN - X1
      F1 = (BOT-A)/SPAN - X1/ESPAN
      F2 = (TOP-A)/SPAN - XN/ESPAN
      RETURN
      END
```

```
      FUNCTION EXPT(X)
      Y = MAX(X,-20.0)
      Y = MIN(Y,20.0)
      EXPT = EXP(Y)
      RETURN
      END
```

```
      SUBROUTINE FCN2(X,F,IDUM,IEST)
      REAL X(2),F(2)
      REAL ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BNDS/ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      REAL A,B,P,Q,FNOW
      COMMON /ESTS/A,B,P,Q,FNOW
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      CALL COUNT(IEST,1)
      GO TO (100,200,200,100,100,300),IEST
100   WRITE(6,77)IEST
77    FORMAT(' -FCN2- ERROR: IEST=',I7)
      RETURN
200   CONTINUE
      P = RESHPE(X(1),PLOW,PHIGH)
      Q = RESHPE(X(2),PLOW,PHIGH)
      CALL ABPQ(IEST,A,B,P,Q)
      CALL M1M2(F1,F2,A,B,P,Q)
      GO TO 9000
300   A = RESHPE(X(1),ALOW,AHIGH)
      B = RESHPE(X(2),BLOW,BHIGH)
      CALL MLAB(F1,F2,A,B,P,Q,N)
9000  CALL PENLTY(F1,X(1))
      CALL PENLTY(F2,X(2))
      F(1) = F1
      F(2) = F2
      FNOW = F1*F1 + F2*F2
      CALL BETTER
      RETURN
      END
```

```
      SUBROUTINE FCN4(X,F,IDUM,IEST)
      REAL X(4),F(4)
      REAL ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BNDS/ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      REAL A,B,P,Q,FNOW
      COMMON /ESTS/A,B,P,Q,FNOW
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      CALL COUNT(IEST,1)
      A = RESHPE(X(1),ALOW,AHIGH)
      B = RESHPE(X(2),BLOW,BHIGH)
      P = RESHPE(X(3),PLOW,PHIGH)
      Q = RESHPE(X(4),PLOW,PHIGH)
      GO TO (100,200,300,400,500),IEST
      WRITE(6,77)IEST
77    FORMAT(' -FCN4- ERROR: IEST=',I7)
      RETURN
100   CONTINUE
      CALL M1M2(F1,F2,A,B,P,Q)
      CALL A3A4(F3,F4,P,Q)
      GO TO 9000
200   CALL IXNIX1(F3,F4,A,B,P,Q,N)
      CALL M1M2(F1,F2,A,B,P,Q)
      GO TO 9000
300   CALL EXNEX1(F3,F4,A,B,P,Q,N)
      CALL M1M2(F1,F2,A,B,P,Q)
      GO TO 9000
400   CALL A3A4(F3,F4,P,Q)
      CALL IXNIX1(F1,F2,A,B,P,Q,N)
      GO TO 9000
500   CALL A3A4(F3,F4,P,Q)
      CALL EXNEX1(F1,F2,A,B,P,Q,N)
9000  CALL PENLTY(F1,X(1))
      CALL PENLTY(F2,X(2))
      CALL PENLTY(F3,X(3))
      CALL PENLTY(F4,X(4))
      F(1) = F1
      F(2) = F2
      F(3) = F3
      F(4) = F4
      FNOW = F1*F1+F2*F2+F3*F3+F4*F4
      CALL BETTER
      RETURN
      END
```

```
      SUBROUTINE FINIS(IEST,ARRAY,NMAX)
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL ARRAY(NMAX,6)
      REAL MEANS(6)
      REAL STAT(3,6)
      REAL COR(21)
      INTEGER INCD(21)
      CALL BEMMI(ARRAY,NREPS,6,NMAX,MEANS,
     +     STAT,COR,INCD,IER)
      WRITE(7,210)PTRUE,QTRUE
210   FORMAT(//5X,'TRUE P:',F8.3,
     +    3X,'TRUE Q:',F8.3)
      WRITE(7,215)
      SRN = SQRT(FLOAT(NREPS))
      DO 200 J=1,6
      R = STAT(1,J)
      XM = MEANS(J)
      XMSE = R*R+XM*XM
      STDERR = R/SRN
      WRITE(7,216)XM,XMSE,STDERR,R
200   CONTINUE
      WRITE(7,220)(COR(J),J=1,21)
215   FORMAT(//10X,'MEAN',3X,'MEAN SQUARE',
     +     5X,'STD ERROR',
     +     7X,'STD DEV')
216   FORMAT(4F14.4)
220   FORMAT(/' CORRELATION COEFFICIENTS:'
     +     /F7.3/2F7.3/3F7.3/4F7.3
     +     /5F7.3/6F7.3)
      CALL COUNT(IEST,0)
      RETURN
      END
```

```
      SUBROUTINE FULLM(ARRAY,NMAX)
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      REAL A1,B1,P1,Q1,FN1
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      REAL ARRAY(NMAX,6)
      BONE = A3*A3
      R = 6.0*(A4-BONE-1.0)/(6.0+3.0*BONE-2.0*A4)
      IF (R.LT.0) RETURN
      TEMP=A4*(R+2.0)*(R+3.0)-3.0*(R+1.0)*(R-6.0)
```

```
      IF(TEMP.EQ.0.0) RETURN
      TEMP=1-24.0*(R+1.0)/TEMP
      IF (TEMP.LT.0.0) RETURN
      TEMP=R*SQRT(TEMP)
      IF (A3.LT.0.0) TEMP = -TEMP
      P1 = (R - TEMP)/2.0
      Q1 = P1 + TEMP
      RNGE=R*SDEV*SQRT((R+1.0)/(P1*Q1))
      A1 = M1 - RNGE*P1/R
      B1 = A1 + RNGE
      FN1 = 0.0
      RETURN
      END
```

```
      SUBROUTINE HELPER(J,U,V,X,Y)
      REAL PTRUE,QTRUE
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL U,V,X,Y
      REAL US(12),VS(12),XS(12),YS(12)
      SAVE US,VS,XS,YS
      IF (J.GT.0 .AND. J.LT.13)GO TO 200
      DO 100 K=1,12
      US(K) = (1.0+PT24(K))/2.0
      VS(K) = (1.0-PT24(K))/2.0
      CALL MYBETI(US(K),PTRUE,QTRUE,XS(K),IER)
100   CALL MYBETI(VS(K),PTRUE,QTRUE,YS(K),IER)
      RETURN
200   U = US(J)
      V = VS(J)
      X = XS(J)
      Y = YS(J)
      RETURN
      END
```

```
      SUBROUTINE IXNTX1(F1,F2,A,B,P,Q,N)
      REAL F1,F2,A,B,P,Q,RN
      INTEGER N
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      RN = FLOAT(N)
      A = MIN(A,BOT)
      B = MAX(B,TOP)
      BA = B - A
      CALL MYBETA((TOP-A)/BA,P,Q,F1,IER)
      CALL MYBETA((BOT-A)/BA,P,Q,F2,IER)
      F1 = F1 - RN/(RN+1.0)
      F2 = F2 - 1.0/(RN+1.0)
      RETURN
      END
```

```
      SUBROUTINE LIMITS(IEST)
      REAL A1,B1,P1,Q1,FN1
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      REAL ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BNDS/ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      PLOW = 0.1
      PHIGH = 10.0
      GO TO (100,100,100,200,200,200),IEST
      WRITE(6,77)IEST
77    FORMAT(' -LIMITS- ERROR: IEST=',I7)
      RETURN
100   ALOW = BOT-9.0*SPAN
      AHIGH = BOT
      BLOW = TOP
      BHIGH = TOP+9.0*SPAN
      RETURN
200   CONTINUE
      IF (P1.LT.0.0 .OR. Q1.LT.0.0) THEN
            P1 = 1.0
            Q1 = 1.0
            GO TO 100
      ENDIF
      ALOW = BOT - (P1-1.0)*SPAN
      BHIGH = TOP + (Q1-1.0)*SPAN
      RN = FLOAT(N)
```

```
      DENOM = (RN-1.0)*(P1+Q1)-RN+2.0
      AHIGH = BOT - (P1-1.0)*SPAN/DENOM
      BLOW = TOP + (Q1-1.0)*SPAN/DENOM
      RETURN
      END
```

```
      SUBROUTINE LOOPER(IEST,
     +      LIMIT,WASTE,TOL)
      INTEGER IEST,LIMIT,WASTE
      REAL TOL
      REAL A,B,P,Q,FNOW
      COMMON /ESTS/A,B,P,Q,FNOW
      REAL X(2),F(2)
      REAL ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BNDS/ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      INTRVL = 0
      DO 100 K = 1,LIMIT
      X(1) = SHAPE(P,PLOW,PHIGH)
      X(2) = SHAPE(Q,PLOW,PHIGH)
      CALL FCN2(X,F,DUM,IEST)
      IF (FNOW.LT.TOL) RETURN
      IF (INTRVL.GE.WASTE) RETURN
      IF (K.LT.LIMIT)
     +      CALL PQAB(IEST,P,Q,A,B)
100   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MLAB(F1,F2,A,B,P,Q,N)
      REAL F1,F2,A,B,P,Q,RN
      INTEGER N
      PARAMETER (MAXS=20)
      REAL SAMPLE(MAXS)
      COMMON /DATUM/SAMPLE
      RN = FLOAT(N)
      DENOM = TLOG(RN*(P+Q-1.0))
      PD = TLOG(ABS(1.0-P))
      QD = TLOG(ABS(1.0-Q))
      F1 = EXPT(DENOM-PD)
      F2 = EXPT(DENOM-QD)
      IF (P.GE.1.0) F1 = -F1
      IF (Q.GE.1.0) F2 = -F2
      IF ((P+Q) .LT. 1.0) THEN
          F1 = -F1
          F2 = -F2
      ENDIF
      BA = B-A
      DO 100 K = 1,N
      X = SAMPLE(K)
      F1 = F1 + BA/(X-A)
100   F2 = F2 + BA/(B-X)
      RETURN
      END
```

```
      SUBROUTINE MODEST(IEST,ARRAY,NMAX)
      COMMON /PARMS/N,I,NREPS,PTRUE,QTRUE
      REAL ARRAY(NMAX,6)
      REAL A,B,P,Q,FNOW
      COMMON /ESTS/A,B,P,Q,FNOW
      COMMON /BEST/A1,B1,P1,Q1,FN1,INTRVL
      REAL BOT,TOP,SPAN
      COMMON /LMTS/BOT,TOP,SPAN
      REAL ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      COMMON /BNDS/ALOW,AHIGH,BLOW,BHIGH,PLOW,PHIGH
      REAL PME,QME
      SAVE PME,QME
      EXTERNAL FCN2
      EXTERNAL FCN4
      REAL X(4)
      REAL WORK(54)
      PARAMETER (NTRY=9)
      REAL TRIAL(NTRY,2)
```

```
      DATA ((TRIAL(K,L),L=1,2),K=1,NTRY)
     +           /0.5,0.5,
     +            0.5,1.0,
     +            0.5,2.0,
     +            1.0,0.5,
     +            1.0,1.0,
     +            1.0,2.0,
     +            2.0,0.5,
     +            2.0,1.0,
     +            2.0,2.0/
      FN1 = 1.E5
      IER = 0
      GO TO (100,200,200,300,300,400),IEST
      WRITE(6,77)IEST
77    FORMAT(' -MODEST- ERROR: IEST=',I7)
      GO TO 9000
100   A1 = TOP
      B1 = BOT
      CALL FULLM(ARRAY,NMAX)
      IF (A1.LE.BOT .AND. B1.GE.TOP) GO TO 9000
      FN1 = 1.E5
      GO TO 8000
200   CONTINUE
      P = PME
      Q = QME
      CALL FULLM(ARRAY,NMAX)
      CALL LIMITS(IEST)
      CALL LOOPER(IEST,10,2,1.E-4)
      IF (FN1.LT.2.E-2) GO TO 230
      DO 220 K = 1,NTRY
      P = TRIAL(K,1)
      Q = TRIAL(K,2)
      X(1) = SHAPE(P,PLOW,PHIGH)
      X(2) = SHAPE(Q,PLOW,PHIGH)
      CALL FCN2(X,WORK,DUM,IEST)
      IF (FN1.LT.2.E-2) GO TO 230
220   CONTINUE
230   P = P1
      Q = Q1
      CALL LOOPER(IEST,20,2,1.E-4)
      IF (FN1.LT.1.E-4) GO TO 9000
      CALL COUNT(IEST,2)
      X(1) = SHAPE(P1,PLOW,PHIGH)
      X(2) = SHAPE(Q1,PLOW,PHIGH)
      NSIG = 2
      CALL ZSPOW(FCN2,NSIG,2,20,IEST,X,DUM,WORK,IER)
      IF (FN1.LT.1.E-4) GO TO 9000
      GO TO 8000
300   P1 = PME
      Q1 = QME
      IF (P1.GT.0.0 .AND. Q1.GT.0.0) THEN
           CALL ABPQ(IEST,A1,B1,P1,Q1)
```

```
            FN1 = 0.0
            GO TO 9000
      ENDIF
      A1 = BOT - 0.001
      B1 = TOP + 0.001
      GO TO 8000
400   P1 = PME
      Q1 = QME
      CALL ABPQ(5,A1,B1,P1,Q1)
      CALL LIMITS(IEST)
      X(1) = SHAPE(A1,ALOW,AHIGH)
      X(2) = SHAPE(B1,BLOW,BHIGH)
      P = P1
      Q = Q1
      IF (P1.LE.1.0 .OR. Q1.LE.1.0) THEN
            CALL FCN2(X,WORK,DUM,IEST)
            GO TO 9000
      ENDIF
      CALL COUNT(IEST,2)
      NSIG = 3
      CALL ZSPOW(FCN2,NSIG,2,20,IEST,X,DUM,WORK,IER)
      GO TO 9000
8000  CALL LIMITS(IEST)
      CALL COUNT(IEST,3)
      X(1) = SHAPE(A1,ALOW,AHIGH)
      X(2) = SHAPE(B1,BLOW,BHIGH)
      X(3) = SHAPE(P1,PLOW,PHIGH)
      X(4) = SHAPE(Q1,PLOW,PHIGH)
      NSIG = 2
      CALL ZSPOW(FCN4,NSIG,4,20,IEST,X,DUM,WORK,IER)
9000  CALL CHECKR(IEST,A1,B1,P1,Q1,FN1,IER)
      EB = B1-1.0
      EP = P1-PTRUE
      EQ = Q1-QTRUE
      ARRAY(I,1) = A1
      ARRAY(I,2) = EB
      ARRAY(I,3) = EP
      ARRAY(I,4) = EQ
      CALL CVMDST(ARRAY(I,5))
      ARRAY(I,6) = FN1
      IF (IEST.EQ.1) THEN
            PME = P1
            QME = Q1
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE MOMNTS(M1,M2,A3,A4,
     +      A,B,P,Q)
      REAL M1,M2
      PQ = P * Q
      PPQ = P + Q
      PQ1 = PPQ + 1.0
      PQ2 = PQ1 + 1.0
      PQ3 = PQ2 + 1.0
      PQ6 = PPQ - 6.0
      TEMP = (B-A)/PPQ
      M1 = A + TEMP * P
      TEMP = TEMP*TEMP
      M2 = PQ*TEMP/PQ1
      TEMP = SQRT( PQ1/PQ )
      A3 = 2.0*(Q-P)*TEMP/PQ2
      A4 = 3.0*PQ1*(2.0*PPQ*PPQ+
     +    PQ*PQ6)/(PQ*PQ2*PQ3)
      RETURN
      END
```

```
      SUBROUTINE MULLER(A,FA,B,FB,C,FC,J)
      J = 0
      AB = A - B
      CB = C - B
      AC = A - C
      DA = FA - FB
      DC = FC - FB
      DIV = AB*CB*AC
      AN = (CB*DA-AB*DC)/DIV
      CN = (AB*AB*DC-CB*CB*DA)/DIV
      IF (ABS(AN) .GT. 1.E-5) GO TO 500
C     USE THE SECANT METHOD
100   CONTINUE
      IF( (FC*FB) .GE. 0.0) GO TO 200
      D = (B*FC-C*FB)/(FC-FB)
      GO TO 5000
200   CONTINUE
      IF( (FA*FB) .GE. 0.0) RETURN
      D = (A*FB-B*FA)/(FB-FA)
      GO TO 5000
C     USE MULLER'S METHOD.
500   DIS = CN*CN-4.0*AN*FB
      IF (DIS.LT.0.0) GO TO 100
      S=1.0
```

```
      IF (CN .LT. 0.0) S = -S
      D = B + (S*SQRT(DIS)-CN)/(AN+AN)
5000  CONTINUE
      IF (D.LE.A .OR. D.GE.C) RETURN
      IF (B.EQ.D) RETURN
      IF ( (FA*FB) .GE. 0.0) GO TO 5500
      IF (D.GT.B) GO TO 5100
      C = B
      FC = FB
      B = D
      J = 2
      RETURN
5100  C = D
      J = 3
      RETURN
5500  IF ((FB*FC) .GE. 0.0) RETURN
      IF (D.GT.B) GO TO 5600
      A = D
      J = 1
      RETURN
5600  A = B
      FA = FB
      B = D
      J = 2
      RETURN
      END
```

```
      SUBROUTINE MYBETA(X,P,Q,PROB,IER)
      REAL X,P,Q,PROB
      IF (X.LE.0.0) THEN
           PROB = 0.0
           RETURN
      ENDIF
      IF (X.GE.1.0) THEN
           PROB = 1.0
           RETURN
      ENDIF
      CALL MDBETA(X,P,Q,PROB,IER)
      IF (IER.NE.0) WRITE(6,77)X,P,Q,PROB,IER
77    FORMAT(' -MYBETA-',4E12.4,I5)
      RETURN
      END
```

```
      SUBROUTINE MYBETI(PROB,P,Q,B,IER)
      REAL PROB,P,Q
      IER = 0
      B = PROB
      IF (B.EQ.0.0 .OR. B.EQ.1.0)RETURN
      A = 0.0
      FA = -PROB
      C = 1.0
      FC = 1.0-PROB
      B = 0.5
      J = 2
      HALVED = 1.76
      DO 1000 K = 1,10
      DO 1000 L = 1,5
      IF ((C-A) .LT. 1.E-4) RETURN
      IF (J.EQ.1) X = A
      IF (J.EQ.2) X = B
      IF (J.EQ.3) X = C
      CALL MYBETA(X,P,Q,FX,IER)
      FX = FX - PROB
      IF (J.EQ.1) FA = FX
      IF (J.EQ.2) FB = FX
      IF (J.EQ.3) FC = FX
      IF ((FC-FA) .LT. 2.E-8) RETURN
      HALVED = HALVED/1.75
      IF ( ((C-A).LT.HALVED))
     +     GO TO 500
C     IF CONVERGENCE IS SLOW, THEN
C     USE BISECTION.
      IF ((FA*FB) .GT. 0.0) GO TO 400
      C = B
      FC = FB
      B = (A+B)/2.0
      J = 2
      GO TO 1000
400   IF ((FB*FC) .GT. 0.0) RETURN
      A = B
      FA = FB
      B = (B+C)/2.0
      J = 2
      GO TO 1000
C     USE MULLER'S METHOD.
500   CONTINUE
      CALL MULLER(A,FA,B,FB,C,FC,J)
      IF (J.EQ.0) RETURN
1000  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE M1M2(F1,F2,A,B,P,Q)
      REAL F1,F2,A,B,P,Q
      REAL M1P,M2P,DUM1,DUM2
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      CALL MOMNTS(M1P,M2P,DUM1,DUM2,
     +      A,B,P,Q)
      F1 = (M1P-M1)/SDEV
      F2 = (M2P-M2)/SDEV
      RETURN
      END
```

```
      SUBROUTINE PENLTY(F,X)
      REAL F,X,Z
      Z = MAX(0.0, ABS(X)-10.0)
      Z = Z*Z
      IF (F.GE.0.0) THEN
           F = F + Z
      ELSE
           F = F - Z
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE PQAB(IEST,P,Q,A,B)
      REAL M1,M2,A3,A4,SDEV
      COMMON /MMNTS/M1,M2,A3,A4,SDEV
      P1 = (B-M1)*(M1-A)/M2 - 1.0
      P = P1 * (M1-A)/(B-A)
      Q = P1 * (B-M1)/(B-A)
      IF (P.LE.0.0) P = 1.0
      IF (Q.LE.0.0) Q = 1.0
      RETURN
      END
```

```
      FUNCTION PT24(K)
      REAL X(12)
      INTEGER K
      DATA X/.9951872200,
     +       .9747285560,
     +       .9382745520,
     +       .8864155270,
     +       .8200019860,
     +       .7401241916,
     +       .6480936519,
     +       .5454214714,
     +       .4337935076,
     +       .3150426797,
     +       .1911188675,
     +       .0640568929/
      PT24 = X(K)
      RETURN
      END
```

```
      FUNCTION RESHPE(T,LOW,HIGH)
      REAL T,LOW,HIGH
      X = SIN(T)
      RESHPE = X*X*(HIGH-LOW) + LOW
      RETURN
      END
```

```
      FUNCTION SHAPE(X,LOW,HIGH)
      REAL X,LOW,HIGH
      TOL = 0.01*(HIGH-LOW)
      IF (X.GT.(HIGH+TOL) .OR. X.LT.(LOW-TOL)) X = (HIGH+LO
      X = MAX(X,LOW)
      X = MIN(X,HIGH)
      SHAPE = ASIN( SQRT((X-LOW)/(HIGH-LOW)))
      RETURN
      END
```

```
      FUNCTION TLOG(X)
      SAVE ITEST,YMIN,YMAX
      DATA ITEST/0/
      IF (ITEST.GT.0) GO TO 100
      ITEST = 1
      YMIN = EXP(-20.0)
      YMAX = 1.0/YMIN
100   CONTINUE
      Y = MAX(X,YMIN)
      Y = MIN(Y,YMAX)
      TLOG = LOG(Y)
      RETURN
      END
```

```
      FUNCTION WT24(K)
      REAL R(12)
      INTEGER K
      DATA R/.0123412298,
     +       .0285313886,
     +       .0442774388,
     +       .0592985849,
     +       .0733464814,
     +       .0861901615,
     +       .0976186521,
     +       .1074442701,
     +       .1155056681,
```

```
     +        .1216704729,
     +        .1258374563,
     +        .1279381953/
      WT24 = R(K)
      RETURN
      END
```

```
      FUNCTION XNMEAN(N,P,Q)
      REAL P,Q,S,RN,U,V,F1,F2
      SAVE NOLD,POLD,QOLD,FOLD
      DATA NOLD/0/,POLD/0.0/,QOLD/0.0/,FOLD/0.0/
      IF (N.EQ.NOLD .AND. P.EQ.POLD .AND. Q.EQ.QOLD) THEN
           XNMEAN = FOLD
           RETURN
      ENDIF
      S=0.0
      RN = FLOAT(N)
      DO 1000 J=1,12
      CALL HELPER(J,U,V,DUM,DUM)
      CALL MYBETA(U,P,Q,F1,IER)
      CALL MYBETA(V,P,Q,F2,IER)
      F1 = EXPT( RN*TLOG(F1) )
      F2 = EXPT( RN*TLOG(F2) )
      S = S + WT24(J)*(F1 + F2)
1000  CONTINUE
      XNMEAN = 1.0 - S/2.0
      NOLD = N
      POLD = P
      QOLD = Q
      FOLD = XNMEAN
      RETURN
      END
```

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This investigation compared six estimation methods for the parameters of the four parameter Beta distribution. The methods used are the full four-moments estimator, four modified moment estimators, and a modified maximum likelihood estimator.

A closed form solution is derived for the full moment estimator which the author has been unable to find in the literature. The four modified moment estimators are each defined by two moment equations and two equations involving the first and last order statistics. The modified maximum likelihood estimate is defined using the skewness and kurtosis indices to define the shape parameters and then solving for the maximum likelihood estimate of the location parameters, when it exists.

Emphasis is placed on computer application to small samples, n=10 and n=20. Six parent populations are used for sampling purposes, using the six populations (up to symmetry) defined by shape parameter values 0.5, 1, and 2. For sampling purposes, the standard location parameters 0 and 1 are used, but the comparisons should be valid for other location parameters. For each of the twelve sample-size/population combinations, two series of 500 samples each are drawn, for a total of 12,000 samples. All six estimators are applied to each sample. Comparison is done using the logarithm of the Cramer Von Mieses (CVM) unweighted distance between the true and estimated distributions.

Emphasis is also placed on solution techniques and on numerical problems, with descriptions of the way these were handled. The raw computer output, the FORTRAN source code, and summaries of the CVM response are shown, and one estimator is identified as superior to the others.

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